A Simple Model of a Central Bank Digital Currency*

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Abstract

We develop a general equilibrium model that highlights the trade-offs between physical and digital forms of retail central bank money. The key differences between cash and central bank digital currency (CBDC) include transaction efficiency, possibilities for tax evasion, and, potentially, nominal rates of return. We establish conditions under which cash and CBDC can co-exist and show how government policies can influence relative holdings of cash, CBDC, and other assets. We illustrate how a CBDC can facilitate negative nominal interest rates and helicopter drops, and also how a CBDC can be structured to prevent capital flight from other assets.

Keywords: Central bank digital currency, cash, medium of exchange, store of value, transaction efficiency.

JEL Classification Numbers: E4, E5, E61.

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1. Introduction

Many central banks around the world are considering or already experimenting with the issuance of retail central bank digital currencies (CBDCs), digital versions of their fiat currencies. The Bahamas, the Eastern Caribbean Currency Union, and Nigeria have already introduced CBDCs. CBDC experiments and pilots are underway in many other economies including China, India, Japan, and Sweden.[1] This concept has gained traction as the use of physical cash (currency banknotes and coins) is declining around the world with digital payment technologies gaining prevalence. There are a few major economies such as Japan, Switzerland, and the United States, where cash remains an important (although increasingly less important) payment mechanism for low-value retail transactions. But in most other advanced and emerging market economies, the proliferation of innovative low-cost retail payment systems is rapidly displacing cash.

A retail CBDC would in principle be available to all agents in an economy, as distinct from the electronic balances (reserves) that financial institutions have access to at central banks. The principal motivations for central banks to consider issuing CBDC appear to be two-fold. In some developing economies, a key objective is to increase financial inclusion by giving households easy and low-cost access to an electronic form of payment. In other countries, the objective is to create a central bank-managed backstop to a payment system and infrastructure managed entirely by the private sector, which could be vulnerable to confidence shocks.

Our focus in this paper is on account-based (also referred to as register-based) retail CBDC, which could be backed up by a payment technology that provides instantaneous and low-cost settlement. CBDC that can be held in digital wallets that, in effect, serve as

[1]For updates on the status of CBDC trials and implementation around the world, please see the following websites: https://www.atlanticcouncil.org/cbdctracker/ and https://kiffmeister.com/category/special/. For evidence on the declining use of cash (and its continued prevalence in some major economies, see Prasad (2021). For a discussion of design issues concerning validation, finality of settlement, anonymity, and security of CBDC transactions, see Allen et al. (2020).

CBDC accounts are becoming the norm in CBDC experiments and proofs-of-concept that are being undertaken in different countries. Cash and CBDC share some similar features–they would both remain liabilities of the central bank but could be distributed and circulated by commercial banks and other payment providers. Rogoff (2017) highlights the differences between cash and CBDC. He argues that a CBDC makes it easier to limit the use of central bank money for illicit activities and to relax the zero lower bound constraint on nominal interest rates. The emergence of CBDC could also have implications for financial inclusion, financial stability, and monetary policy implementation and transmission.

We develop a framework that highlights the economic trade-offs between cash and CBDC. In our model, agents have access to a broad set of assets that they can use for intertemporal consumption smoothing: cash, CBDC, government bonds, and physical capital. The first two are necessary for consumption transactions, i.e., they serve as mediums of exchange. Our model attempts to capture the key attributes that differentiate the two forms of retail central bank money. We model CBDC as providing greater transaction efficiency (lower transaction costs) relative to cash. However, using cash makes it possible to avoid taxes; by contrast, CBDC holdings used for purchases of consumption goods are subject to a tax. Cash provides a zero nominal rate of return while CBDC can have a variable nominal interest rate that can be either positive or negative.

Our model establishes conditions under which the two types of currency issued by a central bank can co-exist and indicates how government policies can affect their relative attractiveness. We show that the share of CBDC in total assets is positively related to the rate of return on CBDC, the relative transaction efficiency of CBDC, degree of monitoring of transactions, and penalties for tax evasion. We also examine the differential sensitivity of the CBDC share to changes in various policy parameters. Our model suggests that the highest degree of sensitivity of the CBDC share is to the nominal rate of return on CBDC. Additionally, we evaluate the welfare implications of different settings of the policy parameters. Increases in the transaction efficiency and rate of return on CBDC generate significant welfare gains.

One of the key attractions of a CBDC is that it allows the central bank, when facing a situation of economic or financial crisis, to impose a negative nominal interest rate on outside money, rather than being constrained by the zero bound on nominal interest rates. In our model, CBDC demand does not collapse to zero even with a negative nominal interest rate because CBDC serves as a more efficient medium of exchange than cash. We also find that negative and positive rates of return on CBDC result in asymmetric effects on allocations across different assets. We show how the probability of helicopter drops of money (as a countercyclical policy tool in the face of a deep recession), which in our model amount to direct transfers into CBDC accounts rather than money-financed fiscal stimulus, can influence the holdings of CBDC relative to other assets. In another extension, we show how the government can encourage CBDC holdings by imposing a tax on cash holdings above a predetermined threshold, a proxy for various policies adopted by countries such as Greece and India to limit the transaction use of cash for tax evasion.

A key concern about a retail CBDC is that it could disintermediate the banking system through capital flight away from bank deposits into CBDC, thereby precipitating financial instability. Our model does not incorporate bank deposits but we show how the rate of return on CBDC can be structured ex-ante to avoid flight from other assets such as bonds, essentially by setting a fee on CBDC holdings above a threshold level.

One question raised by our analysis is why, if a CBDC has lower transaction costs and lower levels of tax evasion than cash, the government would not simply eliminate cash. Even proponents of CBDC note that there are privacy concerns and other political considerations that make it infeasible or, at a minimum, unwise for a government to do away with cash altogether. For instance, in a recent report the BIS lays out a "foundational principle" that a CBDC would need to co-exist with and complement existing forms of money (BIS, 2020). There are also concerns that elimination of cash could disadvantage the poor, who are more prone to exclusion from the formal financial system and have lower access to electronic means of payment. Our analysis could be viewed as showing how the government could, through its policies, endogenously reduce consumers' preference for cash rather than doing this through a directive. The analysis also provides a quantitative assessment of how different factors could affect the evolution of the relative shares of cash and CBDC in an economy.

Our paper abstracts from issues of competition between fiat currencies and private decentralized cryptocurrencies such as Bitcoin. However, by shedding light on the relative importance of a currency's attributes in multiple dimensions, the framework that we develop in the paper has the potential to be extended to study the co-existence of multiple forms of physical and electronic currencies, both official and private.[2]

1.1. Related literature

Digital central bank money has already existed for a long time. Electronic balances held by commercial banks (and certain other financial institutions) at central banks, referred to as reserves, facilitate payments and settlement through interbank payment systems managed by the central bank. Kumhof and Noone (2021) distinguish CBDC from reserves and cash by defining it as electronic central bank money that (i) can be accessed more broadly than reserves, (ii) has functionality for retail transactions, (iii) can be interest bearing, and (iv) has a separate operational structure relative to other forms of central bank money. Yao (2018), former head of the Institute of Digital Money at the People's Bank of China, refers to a CBDC as "a credit-based currency in terms of value, a cryptocurrency from a technical perspective, an algorithm-based currency in terms of implementation, and a smart currency in application scenarios." Auer, Cornelli, and Frost (2020) and Lovejoy et al. (2022) assess the drivers, technical designs, and architecture of different variants of CBDC.[3]

A number of papers have modeled specific differences between cash and CBDC. Agur, Ari, and Dell'Ariccia (2022) model the difference between cash and CBDC as hinging on two

[2] Auer and Claessens (2018, 2021) discuss cryptocurrency regulation.

[3] For further discussion, see BIS (2018), Brunnermier, James, and Landau (2019), and Allen et al. (2020). Gnan and Masciandaro (2018) and Keister and Sanches (2023) discuss the rationale for a CBDC.

features–anonymity and security. Garratt and Lee (2021), Garratt and van Oordt (2021) and Ahnert, Hoffman, and Monnet (2022) discuss the implications of digital payments for privacy. Our contribution to this literature is to incorporate other features that distinguish CBDC from cash, and also to illustrate some design features that are under discussion in ongoing CBDC experiments.

Another topic that has received attention is how a CBDC can be positioned within an array of other payment options and the factors that determine the relative demand for each option. A useful framework for this exercise is provided by Kahn and Roberds (2009), who highlight the essential function of payments and provide a taxonomy of alternative forms of payments, as well as their financial and macroeconomic implications. Bijlsma et al. (2021) and Li (2022) argue that the adoption rate of CBDC increases with the interest rate it bears, the extent of public knowledge about it, and the degree of trust in the banking system.

An important concern is whether a CBDC would disintermediate the banking system. In early contributions, Bjerg (2017) and Bordo and Levin (2017) discuss alternative designs for a CBDC in terms of whether or not it would be interest-bearing and be complementary to or directly compete with bank deposits. Andolfatto (2021) studies the implications of CBDC in an overlapping generations model with a monopolistic banking sector. In this model, the introduction of interest-bearing CBDC increases the market deposit rate, leads to an expansion of the deposit base, and reduces bank profits. However, the CBDC has no effect in terms of bank lending activity and lending rates. Chiu et al. (2022) reach a similar conclusion.

By contrast, Whited, Wu, and Xiao (2022), using a model with frictions that bind deposits and lending, argue that introduction of a CBDC could modestly reduce bank lending. The effect is mitigated by the ability of banks to replace deposits with wholesale funding. In a different setting, Nyffenegger (2022) also finds a modest degree of bank disintermediation when a CBDC is introduced. Fernandez-Villaverde et al. (2020) show how, in an economy with CBDC, depositors can internalize the relative stability of the central bank relative to commercial banks, leading to the central bank becoming a deposit monopolist even in normal times. Williamson (2022) argues that a CBDC can increase welfare by shifting safe assets from the private banking sector to what is effectively a narrow banking facility, which allows for more efficient use of the aggregate stock of safe collateral.

Recent papers have begun to grapple with the implications of CBDC for financial markets and monetary policy.[4] Some authors argue that a CBDC will not materially affect the implementation of monetary policy, although there could be other macroeconomic effects. The conclusions depend on the model structure and the manner in which the CBDC is introduced into the economy. Barrdear and Kumhof (2022) develop a DSGE model with multiple sectors and several nominal and real rigidities. In a model calibrated to U.S. data, they find that infusing CBDC on the order of 30 percent of GDP through central bank purchases of government bonds could result in substantial steady state output gains of nearly 3 percent due to reductions in real interest rates, distortionary taxes, and monetary transaction costs. Bordo and Levin (2017) conclude that a CBDC could bolster the effectiveness of monetary policy and enhance the stability of the financial system. Davoodalhosseini (2022) finds that an interest-bearing CBDC environment can help the central bank attain a more efficient allocation than with cash. Our contribution to this literature is to explicitly model how the government and central bank can affect the choice of cash versus CBDC and what the welfare implications are.

The literature on CBDC of course draws on an extensive literature about models of money. Early approaches include money in the utility function (Sidrauski, 1967), cash-in advance models (Svensson, 1985), shopping-time models (Brock, 1990), and the turnpike model of spatially separated agents (Townsend, 1980). Search-theoretic models of money pioneered by Kiyotaki and Wright (1993) represent a major step forward in this literature. Kocherlakota (1998) highlights the role played by money in environments with incomplete information and limited commitment.

[4] Cong and Mayer (2022) and Ferrari Minesso, Mehl, and Stracca (2022) discuss the cross-border implications of retail CBDC introduction.

2. Model

In this section, we sketch the main features of our model. We focus the exposition on the main differences between cash and CBDC, and show how their relative desirability is affected by key features of the environment, such as relative transaction costs, and also government policy variables such as tax rates, the probability of detecting tax evasion, and the penalty for being detected evading taxes.

2.1. Environment

For ease of exposition, we show the representative agent's maximization problem entirely in real terms. The representative agent derives utility from private consumption goods (c_t) and public consumption goods (c_t^g). The utility functions for the private and public consumption goods are assumed to be additive separable; both functions are assumed to be strictly increasing, strictly concave, and satisfy the Inada conditions. CRRA utility function satisfies the following properties and will be used in our analysis. β is the subjective discount rate ($0 < \beta < 1$). The representative agent maximizes the present discounted value of utility by choosing private consumption goods (c_t), public consumption goods (c_t^g), physical capital (k_t), and holdings of bonds (b_t), cash (ca_t), and CBDC (dc_t).

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)] \tag{1}$$

subject to

$$\begin{aligned} c_t + c_t^g + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{2}$$

$$k_t - (1-\delta)k_{t-1} \leq i_t \tag{3}$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{4}$$

In the budget constraint (equation (2)), the LHS denotes the uses of funds while the RHS shows the sources of funds.[5] The available funds are used to purchase private and public consumption goods, undertake investment, purchase bonds, bear transaction costs, pay a penalty for tax evasion, hold cash and central bank digital currency, and pay taxes. The terms τ and m denote the constant tax rate and probability of monitoring due to tax evasion, respectively. π_t denotes the inflation rate.[6] It is assumed that the relative prices of public consumption goods to private consumption goods and of investment to private consumption goods are both one. Note that the subscript *t-1* denotes beginning of period real stocks of assets and the subscript t denotes end of period stocks.

Transaction costs are a function of consumption goods, cash, and CBDC. The properties of the transaction cost function are as follows:[7]

(i) $\phi \geq 0$, $\phi(0, ca, dc) = 0$

(ii) $\phi_c \geq 0$, $\phi_{ca} \leq 0$, $\phi_{dc} \leq 0$

(iii) $\phi_{cc} \geq 0$, $\phi_{caca} \geq 0$, $\phi_{dcdc} \geq 0$, $\phi_{cac} \leq 0$, $\phi_{dcc} \leq 0$

Cash has a higher transaction cost than CBDC, a zero nominal rate of return, and allows for tax evasion.[8] CBDC has a lower transaction cost than cash, could pay interest (which can be positive or negative), and transactions using it are subject to taxation. The agent pays a penalty on tax evasion with a constant probability of being monitored. Thus, the penalty function depends on real cash balances. The penalty function, denoted by $\psi(\frac{ca_{t-1}}{1+\pi_t})$, is assumed to be strictly increasing.[9]

[5] Appendix A.1 shows the derivation of the real budget constraint.

[6] $(1 + \pi_t) = \frac{P_t}{P_{t-1}}$, where P_t is the price level in period t.

[7] Feenstra (1986) has a similar set of assumptions on the transaction cost function. Fried and Howitt (1983) propose an early model in which both government bonds and cash provide liquidity functions. Our model can be thought of as introducing an additional asset that has both these features while bonds cannot be directly used for transactions.

[8] Alternatively, we could model cash as providing the benefit of privacy rather than facilitating tax evasion, although this would require us to include the utility benefits from privacy in the model.

[9] We maintain the generality of the model at this stage and describe the equilibrium and steady state conditions before specifying particular functional forms for the utility, production, transaction cost, and tax evasion penalty functions in Section 2.4.

The source of funds in a given period is given by the sum of real income from output, interest payments accrued from bond holdings, interest payments (if any) from CBDC, and cash holdings from the previous period. Output in the economy equals $f(k_{t-1})$. Physical capital is the only factor of production; the production function $f(k_{t-1})$ is strictly increasing, strictly concave, and satisfies the Inada conditions. I_t^b and I_t^d denote the gross nominal interest rates on bond holdings and on CBDC, respectively.[10]

Equation (3) represents the evolution of the stock of physical capital, with δ being the depreciation rate. Equation (4) denotes a liquidity constraint–the purchase of private consumption goods cannot exceed the amount of cash holdings and CBDC holdings at the beginning of the period.[11] The state variables are physical capital (k_{t-1}), bond holdings (b_{t-1}), real cash balances (ca_{t-1}), and real CBDC (dc_{t-1}).[12]

The government consumes g_t units of output each period and produces η units of public goods per unit consumed.[13] Thus, equation (5) represents the government provision of public consumption goods.

$$c_t^g = \eta g_t \tag{5}$$

The consolidated government budget constraint is given by equation (6). The LHS and RHS represent the government's expenditure and income, respectively. The government spends on own expenditure net of provision of public goods and pays returns on bond and CBDC holdings. The government obtains revenue from issuing bonds, seigniorage from

[10] $I^b = R^b\Pi$, $I^d = R^d\Pi$, and $\Pi = (1 + \pi)$. R^b, R^d represent the gross real interest rates on bonds and CBDC. $I^b = 1+i^b$, $I^d = 1+i^d$, $R^b = 1+r^b$, and $R^d = 1+r^d$, where i^b, i^d, r^b, and r^d represent net nominal interest rate on bond holdings, net nominal interest rate on CBDC, net real interest rate on bond holdings, and net real interest rate on CBDC, respectively.

[11] Appendix A.1 shows the derivation of the real liquidity constraint. The liquidity constraint does not bind in equilibrium, implying that agents always have sufficient real balances in the form of cash and CBDC to obtain consumption goods. The liquidity constraint and the transaction function together rule out the possibility of bonds serving as a medium of exchange.

[12] An endowment economy with no physical capital would be a special case of our model (see section 4.7.3). We incorporate physical capital to have a more generalizable model that includes a relatively safe asset (bonds) and a riskier one (physical capital), although we do not explicitly model the risky return from physical capital in the present paper.

[13] Setting the parameter $\eta = 1$ implies that the government does not waste any resources. Thus, $g_t - c_t^g$ represents the administrative costs of running the government that do not benefit the representative agent.

issuing CBDC and cash (both of which have a zero cost of issuance), taxation on consumption transactions using CBDC, and penalties collected from tax evaders.

$$\begin{aligned} & g_t - c_t^g + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \\ & = b_t + dc_t + ca_t + \frac{\tau}{1+\pi_t} dc_{t-1} + m\psi(\frac{ca_{t-1}}{1+\pi_t}) \end{aligned} \tag{6}$$

The monetary authority controls the circulation of money supply (sum of cash and CBDC). We assume that the nominal stock of cash and CBDC grows at an exogenous and constant rate μ. We use upper case variables CA_t, DC_t, and P_t to denote nominal cash, nominal CBDC, and the price level, respectively.

$$CA_t + DC_t = (1+\mu)(CA_{t-1} + DC_{t-1}) \tag{7}$$

The real stock of money supply grows at a rate given by μ minus the rate of inflation.

$$ca_t + dc_t = \frac{1+\mu}{1+\pi_t}(ca_{t-1} + dc_{t-1}) \tag{8}$$

The change in real money supply, given by equation (9), depends on the inflation rate.

$$\Delta(ca_t + dc_t) = \Delta(\frac{CA_t + DC_t}{P_t}) = \frac{CA_t + DC_t}{P_t} - \frac{CA_{t-1} + DC_{t-1}}{P_{t-1}} = \frac{\mu - \pi_t}{1+\pi_t}(ca_{t-1} + dc_{t-1}) \tag{9}$$

2.2. Equilibrium

Solving the optimization problem by using the Bellman equation yields the following intertemporal conditions. The detailed solution using the dynamic optimization method is shown in Appendix A.2.

$$\frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = v_{c^g}(c_t^g) \tag{10}$$

$$\frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = \beta \frac{u_c(c_{t+1})}{1 + \phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}(f_k(k_t) + 1 - \delta) \tag{11}$$

$$\frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = \beta \frac{u_c(c_{t+1})}{1 + \phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \frac{I_t^b}{1+\pi_{t+1}} \tag{12}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} =&\beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}\\ &(\frac{1}{1+\pi_{t+1}}-\frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}-\frac{\phi_{ca}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} =&\beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}\\ &(\frac{I^d_t}{1+\pi_{t+1}}-\frac{\tau}{1+\pi_{t+1}}-\frac{\phi_{dc}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{14}$$

The optimal paths of c_t, c^g_t, b_t,k_t, ca_t, and dc_t are characterized by equations (10)-(14), the budget constraint of the representative agent by equations (2)-(4), the provision of public consumption goods by equation (5), the government budget constraint by equation (6), and the change in real cash balances by equation (9).[14] Equation (10) represents the intratemporal relationship between private and public consumption goods. Equations (11)-(14) represent the Euler equations for capital, bonds, cash, and CBDC, respectively.

2.3. Steady state

The key equations that characterize the steady state of the economy are as follows:

$$\frac{u_c(\bar{c})}{1+\phi_c(\bar{c},\frac{\bar{ca}}{1+\bar{\pi}},\frac{\bar{dc}}{1+\bar{\pi}})}=v_{c^g}(\bar{c^g}) \tag{15}$$

$$1=\beta(f_k(\bar{k})+1-\delta) \tag{16}$$

$$1=\beta\frac{\bar{I}^b}{1+\bar{\pi}} \tag{17}$$

$$1=\beta(\frac{1}{1+\bar{\pi}}-\frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}}-\frac{\phi_{ca}(\bar{c},\frac{\bar{ca}}{1+\bar{\pi}},\frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \tag{18}$$

$$1=\beta(\frac{\bar{I}^d}{1+\bar{\pi}}-\frac{\tau}{1+\bar{\pi}}-\frac{\phi_{dc}(\bar{c},\frac{\bar{ca}}{1+\bar{\pi}},\frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \tag{19}$$

$$\begin{aligned}\bar{c}+\bar{c^g}+\delta\bar{k}+\bar{b}+\phi(\bar{c},\frac{\bar{ca}}{1+\bar{\pi}},\frac{\bar{dc}}{1+\bar{\pi}})+m\psi(\frac{\bar{ca}}{1+\bar{\pi}})+\bar{ca}+\bar{dc}+\frac{\tau}{1+\bar{\pi}}\bar{dc}\\ =f(\bar{k})+\frac{\bar{I}^b}{1+\bar{\pi}}\bar{b}+\frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc}+\frac{\bar{ca}}{1+\bar{\pi}}\end{aligned} \tag{20}$$

[14] The liquidity constraint (equation(4)) doesn't bind in equilibrium. Thus, the Lagrange multiplier associated with this inequality is zero according to the complementary slackness condition.

$$(\bar{g} - \bar{c^g}) + \frac{\bar{I^b}}{1+\bar{\pi}}\bar{b} + \frac{\bar{I^d}}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} = \bar{b} + \bar{dc} + \bar{ca} + \frac{\tau}{1+\bar{\pi}}\bar{dc} + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) \tag{21}$$

$$\bar{c^g} = \eta\bar{g} \tag{22}$$

$$0 = \Delta(\bar{ca} + \bar{dc}) = \frac{\mu - \bar{\pi}}{1+\bar{\pi}}(\bar{ca} + \bar{dc}) \tag{23}$$

$\bar{k}$, $\bar{c}$, $\bar{g}$, $\bar{c^g}$, $\bar{ca}$, $\bar{dc}$, $\bar{b}$, $\bar{I^b}$, and $\bar{\pi}$ are determined from the above steady-state conditions.

2.4. Analysis

We assume the following functional forms for the utility function, the production function, the transaction cost function, and the tax evasion penalty function.

$$u(c) = \frac{c^{1-\epsilon_c}}{1-\epsilon_c} \tag{24}$$

$$v(c^g) = \frac{(c^g)^{1-\epsilon_g}}{1-\epsilon_g} \tag{25}$$

$$f(k) = Ak^{\alpha} \tag{26}$$

where $0 < \alpha < 1$ and A is productivity.

$$\phi(c, ca, dc) = \theta_1 \frac{c^a}{ca^{\gamma} dc^{\rho}} \tag{27}$$

where $a > 1$, $\gamma > 0$, $\rho > 0$, and $\gamma < \rho$

$$\psi(ca) = \theta_2 ca^{\nu} \tag{28}$$

where $\theta_2 > 1$ and $\nu > 1$

We have kept the structure of these functions simple to the extent possible and such that they meet the general conditions on each of the functions mentioned in Section 2.1. The transaction cost function allows for an interior solution with cash and CBDC co-existing as mediums of exchange.[15] The structure of this function allows us to easily evaluate the effects

[15]This is in the spirit of what central banks are considering, i.e., the co-existence of cash and CBDC, at least for the foreseeable future.

of changes in the relative transaction efficiency of cash and CBDC. We also incorporate a non-negativity constraint on bond holdings; this constraint is not binding for most of our parameter settings (we discuss some exceptions to this later in the paper).

The detailed solution for the steady state is provided in Appendix A.3. Solving for the above functional form yields the following for $\bar{k}$, $\bar{c}$, $\bar{g}$, $\bar{c^g}$, $\bar{ca}$, $\bar{dc}$, $\bar{b}$, $\bar{I^b}$,and $\bar{\pi}$. The steady state value of physical capital is given by equation (29).

$$\bar{k} = (\frac{A\alpha\beta}{1-\beta(1-\delta)})^{\frac{1}{1-\alpha}} = \Theta_k(A, \alpha, \beta, \delta) \tag{29}$$

The inflation rate in steady state is derived from the condition of no change in real money supply in the economy (equation (23)). Hence, the inflation rate is equal to the growth rate of cash and CBDC.

$$\bar{\pi} = \mu = \Theta_\pi(\mu) \tag{30}$$

The steady state value of the gross nominal interest rate of bonds is given by equation (31).

$$\bar{I^b} = \frac{1+\bar{\pi}}{\beta} = \frac{1+\mu}{\beta} = \Theta_{I^b}(\beta, \mu) \tag{31}$$

The steady state levels of private and public consumption goods and private consumption goods are obtained by substituting equation (21) into equation (20).

$$\bar{c} + \bar{g} + \phi(\bar{c}, \bar{ca}, \bar{dc}) = f(\bar{k}) - \delta\bar{k} = \bar{k}^\alpha - \delta\bar{k} = \Theta_3(A, \alpha, \beta, \delta) \tag{32}$$

The government sets the nominal interest rate of CBDC in steady state. Using the intratemporal relationship between private and government public consumption goods, the intertemporal equation for cash holdings, and the intertemporal equation for CBDC holdings, one can determine the levels of cash, CBDC, private consumption goods, public consumption goods, and government expenditure as functions of the model primitives. The level of bond holdings in the steady state is determined from the government's consolidated budget constraint.

3. Results

We now conduct five types of experiments using the model. First, we evaluate the impacts of changes in specific policy parameters. Second, we present alternative baseline scenarios as a robustness test for our main conclusions. Third, we analyze the effects of varying a particular parameter over a range of possible values. Fourth, we provide a welfare comparison of alternative policy choices. Fifth, we explore the effects of imposing a negative nominal interest rate on CBDC. These are all meant to be illustrative exercises and, since there are no major economies operating with retail CBDC, at this stage we do not attempt to match any specific data moments.

The results of the quantitative exercises we undertake are described in terms of the composition of asset holdings of the representative agent. These assets are composed of physical capital and holdings of bonds, cash, and CBDC. The value of total assets in steady state ($\bar{fa}$) can be defined as the sum of the steady state values of these four assets.

$$\bar{fa} = \bar{k} + \bar{b} + \bar{ca} + \bar{dc}$$

We examine the share of each asset in total assets in the steady states implied by different parameter settings. For instance, the share of CBDC in total assets is given by

$$\bar{dc}^{share} = \frac{\bar{dc}}{\bar{fa}}$$

The parameters of the model influence the steady state values of physical capital, private consumption goods, public consumption goods, cash holdings, CBDC holdings, bond holdings, the rate of return on bonds, and the inflation rate. As our main objective is to understand the trade-offs between holdings of cash and CBDC, we restrict our analysis to parameters that influence these holdings. These parameters are γ, ρ, m, θ_2, τ, and $\bar{I}^d$. The influence of these parameters on the choice variables is determined by the intertemporal equations for cash holdings and CBDC holdings, the consolidated budget constraint, and the intratemporal equation for private and public consumption goods. Note that these parameters have no effect on the steady state values of capital, the nominal rate of return on bonds, and the inflation rate.

The parameter settings for the baseline model (BM) for determining the steady state are shown in Table 1. We adopt conventional values for standard parameters and, where feasible, rely on empirical observations for others. For instance, the tax rate parameter is set at 0.08 based on the average state sales tax in the U.S., which is roughly 8 percent. As noted above, our objective is to conduct some illustrative exercises rather than match any specific data moments. Hence, for the remaining parameters for which there are no clear analogues in the data (as no economy has a full-fledged CBDC as yet), we pick arbitrary values and then attempt to provide some economic intuition about how varying those values affects the dynamics of the model.[16] The time period in our model is equivalent to one year and we pick parameters corresponding to this frequency.

We consider six alternative parameter settings, in each of which we vary one parameter relative to the BM. Each of these changes can be viewed as a government policy intervention to increase the representative agent's relative holdings of CBDC. Alternative model-I (AM-I), with a lower γ parameter, corresponds to a decrease in the transaction efficiency of cash. Alternative model-II (AM-II) corresponds to an increase in the transaction efficiency for CBDC (higher value for ρ); alternative model-III (AM-III) corresponds to an increase in the rate of monitoring, i.e., a higher probability of detection of tax evasion (higher m); alternative model-IV (AM-IV) corresponds to an increase in the penalty for tax evasion (higher θ_2); alternative model-V (AM-V) corresponds to a decrease in the tax rate (lower τ); and alternative model-VI (AM-VI) corresponds to an increase in the rate of return on CBDC (higher $\bar{I}^d$).

[16]Some papers contain indirect estimates of the relative efficiency of digital payments and the demand for CBDC but these are difficult to map into our model. Biljsma et al. (2021), Li (2022), and Whited, Wu, and Xiao (2022) contain estimates of the digital premium and the demand for CBDC. These are not easy to compare even relative to each other and are sensitive to the model specification. Our sensitivity experiments cover a range of possibilities for the relative transaction efficiency of CBDC compared to cash.

Table 1: Parameters for the baseline model

Parameter	Description	
β	Discount factor	0.95
η	Proportion of government spending on public good	0.80
ϵ_c	Inverse of IES for private consumption goods	2.00
ϵ_g	Inverse of IES for public consumption goods	1.50
m	Probability of monitoring	0.15
τ	Tax rate	0.08
$\bar{I}_d$	Steady state nominal rate of return on CBDC	1.05
μ	Growth rate of money supply	0.01
α	Capital share	0.33
A	Productivity	1.00
δ	Depreciation rate	0.10
θ_1	Level parameter of transaction function	2.00
a	Transaction costs for consumption goods	2.00
γ	Transaction efficiency for cash	1.05
ρ	Transaction efficiency for CBDC	1.75
θ_2	Level parameter of penalty function	2.00
ν	Sensitivity to cash in penalty function	3.00

Notes: Conventional values for β, ϵ_c, α, and δ. τ set at 8% as the average U.S. state sales tax is around 8%. ϵ_g is set lower than ϵ_c. μ is set at 1% to have an inflation rate at 1%. γ is set at 0.6 times ρ. Other parameters are set arbitrarily.

3.1. Comparisons across policies

To standardize the changes in the policy parameters, each alternative model involves a 1 percent change in the value of the relevant parameter relative to its baseline value shown in Table 1. Table 2 shows the shares of the different assets in steady state for each parameter setting.

Table 2: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	37.44	37.46	37.28	37.41	37.41	37.53	38.70
Bond holdings	16.80	16.73	16.88	16.84	16.84	16.38	10.67
Cash holdings	6.48	6.46	6.45	6.46	6.46	6.49	6.60
CBDC holdings	39.28	39.35	39.39	39.29	39.29	39.61	44.03
CBDC (% change)	-	0.17	0.27	0.03	0.03	0.83	12.08

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

There are two important points to note in the results in Table 2. First, both cash and CBDC co-exist in the economy under both baseline and alternative parameter settings. Second, the representative agent holds a low share of cash in all of these models, consistent with the shift toward a near cashless economy that many developed and developing economies seem to be experiencing. The elasticity of the CBDC share with respect to the parameters in the alternative model varies widely. For example, the elasticity of the CBDC share with respect to the rate of return on CBDC is 12.08. This implies that a one percent increase in the rate of return on CBDC results in a 12.08 percent increase in the CBDC share. The elasticity of the CBDC share with respect to changes in the tax rate is lower. A one percent decrease in the tax rate results in an increase of 0.83 percent in the CBDC share.

The share of CBDC in total financial assets increases significantly for four policies and barely changes for the other two policies. The share of CBDC in total assets can change through direct channels that affect CBDC holdings or through indirect channels that affect cash holdings. The direct channels include increases in the transaction efficiency of CBDC, which could be affected by policy measures, as well as more explicit policy interventions such as a decrease in the tax rate or increase in the nominal rate of return on CBDC. Agents increase their share of CBDC in total assets when it becomes a more lucrative store of value or a more efficient medium of exchange.

The indirect channels include a decrease in the transaction efficiency of cash and policy interventions such as an increase in the probability of monitoring and an increase in the penalty for being caught evading taxes. Interestingly, not all of these indirect channels result in the expected change in the CBDC share of total assets. When cash becomes a less efficient medium of exchange, the CBDC share rises. However, the other two indirect channels related to tax evasion have virtually no effect on the CBDC share. This is perhaps because, while these channels directly impact the tax evasion role of cash, they have no effect on the medium of exchange or store of value aspects of CBDC.

Among these policy changes, an increase in the nominal rate of return on CBDC has the largest impact on CBDC holdings. Table 2 yields the following ordering in terms of the effects of different policy changes on CBDC shares: increase in rate of return on CBDC $\succ$ decrease in tax rate $\succ$ increase in transaction efficiency of CBDC $\succ$ decrease in transaction efficiency of cash $\succ$ increase in the probability of monitoring $\sim$ increase in the penalty function.

3.2. Alternative baselines

A significant challenge we face is the lack of empirical data even from CBDC experiments and pilots to discipline the key policy parameters in our model. Hence, we now turn to an alternative approach by evaluating the robustness of our results to different combinations of the main policy parameters. Table 3 shows five of these combinations and the associated steady state shares of various assets. This table confirms that the relative shares of different assets, and the effects of changes in policy parameters on those shares, are not sensitive to the specific baseline that we assume. This is further confirmed by an extensive battery of experiments we conducted with sensitivity tests that involve changing specific parameters around alternative baselines (these results are shown in Appendix A.5). For instance, reducing CBDC transaction costs and increasing the tax evasion penalty increases the share of CBDC across alternative settings of most other parameters, while reducing the rate of return on CBDC has the opposite effect. This table does illustrate interesting interactions among various parameter settings. For example, the share of CBDC holdings rises when the transaction efficiency of CBDC is higher even if the rate of return on CBDC is reduced (comparing column BM with columns BM-I, BM-II, BM-IV, and BM-V). These alternative calibrations might prove useful when data from CBDC trials become available and make it possible to pin down at least some of the policy parameters in our model.

Table 3: Parameters and steady state shares (in percent) for baseline models

Description	**BM**	**BM-I**	**BM-II**	**BM-III**	**BM-IV**	**BM-V**
γ	**1.05**	1.50	1.75	2.00	1.00	0.95
ρ	**1.75**	2.00	2.00	2.50	1.50	1.20
m	**0.15**	0.10	0.20	0.30	0.25	0.10
θ_2	**2.00**	3.00	2.50	1.50	2.50	3.00
τ	**0.80**	0.50	0.30	0.40	0.50	0.30
$\bar{I}_d$	**1.05**	1.04	1.03	1.03	1.04	1.02
Physical capital	**37.44**	37.10	36.73	32.22	40.58	46.61
Bond holdings	**16.80**	7.99	2.84	13.21	4.96	0.41
Cash holdings	**6.48**	6.90	6.13	5.73	5.59	7.93
CBDC holdings	**39.28**	48.01	54.30	48.84	48.87	45.05

Notes: BM refers to the baseline model as stated in Table 2. BM-I to BM-V corresponds to five different combinations of policy parameters.

3.3. Asymmetric responses to changes in policy parameters

Next, we examine possible asymmetries and nonlinearities in the shares of CBDC in total assets ($\bar{dc}^{share}$) across different settings of each policy parameter. We show what happens when we vary the value of each relevant parameter over a range from minus to plus 1 percent of its baseline value.

Table 4 shows the change (in percentage points) in the steady state share of CBDC in total assets for the two parameter settings (plus/minus 1 percent) in each alternative model relative to the baseline model. Most of the changes in CBDC shares are relatively modest and symmetric. The highest degree of sensitivity of this share is to the rate of return on CBDC. We explore the effects of this parameter in Figure 1, which varies it across a much broader range. It appears that the relationship between $\bar{dc}^{share}$ and the nominal rate of return on CBDC (I^d_{ss}) is captured well by a cubic function. The asymmetry arises from the fact that, even when its rate of return falls, the CBDC remains useful as a medium of exchange. When the interest rate rises, its store of value function and transactional efficiency reinforce each other and lead to more substitution away from other assets.

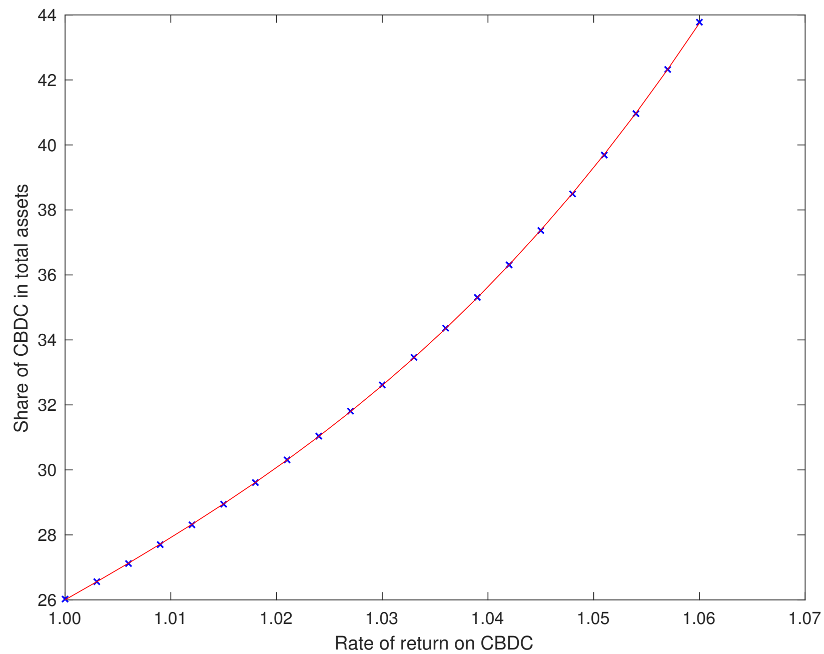
Figure 1 shows that the share of CBDC in total assets is 26.03% when the gross nominal return on CDBC is one. This implies that the net nominal rate of return on CBDC is zero, the same as cash, but CBDC accounts for a larger share of assets than cash (6.06% of total assets) as it is a more efficient medium of exchange.

Table 4: Steady state shares (in percent) for positive and negative changes to policy parameters

	AM-I (γ)		**AM-II** (ρ)		**AM-III** (m)		**AM-IV** (θ_2)		**AM-V** (τ)		**AM-VI**(I^d)	
Variable	**-1%***	**1%**	**-1%**	**1%***	**-1%**	**1%***	**-1%**	**1%***	**-1%***	**1%**	**-1%**	**1%***
CBDC	39.35	39.21	39.18	39.39	39.27	39.29	39.27	39.29	39.60	38.96	35.47	44.03
Deviation	0.07	-0.07	-0.10	0.11	-0.01	0.01	-0.01	0.01	0.32	-0.32	-3.81	4.75

Notes: The results reported in this table are the shares of CBDC in total assets based on positive and negative deviations of specific parameters from their baseline values. The symbol $*$ indicates the results of the experiments reported in Table 2. The share of CBDC in the baseline model is 39.28.

Figure 1: Relationship between the steady state share of CBDC and nominal rate of return of CBDC



Note: This figure shows how the steady state share of CBDC is influenced by the gross nominal rate of return on CBDC, keeping other parameters at their baseline levels. The relationship is well approximated by the following cubic function: $\bar{dc}^{share} = -21887.75 + 65898.21\bar{I}^{d} - 66239.27\bar{I}^{d2} + 22254.81\bar{I}^{d3}$

3.4. Welfare comparison

One normative question raised by our analysis is whether a benevolent government should attempt to raise the relative share of CBDC through its policy choices. To answer this question requires an evaluation of welfare outcomes under different policy settings. While recognizing that our model abstracts from many advantages of cash, especially in economies in which the population has limited access to the formal financial system, we now turn to such an evaluation using our model.

We define welfare in the baseline model as the discounted sum of the present and future utility streams of the representative agent:

$$V_0^b = \sum_{t=0}^{\infty} \beta^t (u(c_t^b) + v(c_t^{gb}))$$

Let ω^a represent the conditional welfare gain from adopting an alternative model. That is, ω^a denotes the fraction of private consumption goods that have to be added under a particular alternative model to achieve the same welfare as in the baseline model (Schmitt-Grohé and Uribe, 2007). As before, each alternative model that we analyze involves changing one parameter and keeping all other parameters the same as in the baseline model. Welfare in the alternative model is given by:

$$V_0^a = \sum_{t=0}^{\infty} \beta^t (u(c_t^a) + v(c_t^{ga})) = \sum_{t=0}^{\infty} \beta^t (u((1+\omega^a)c_t^b) + v(c_t^{gb}))$$

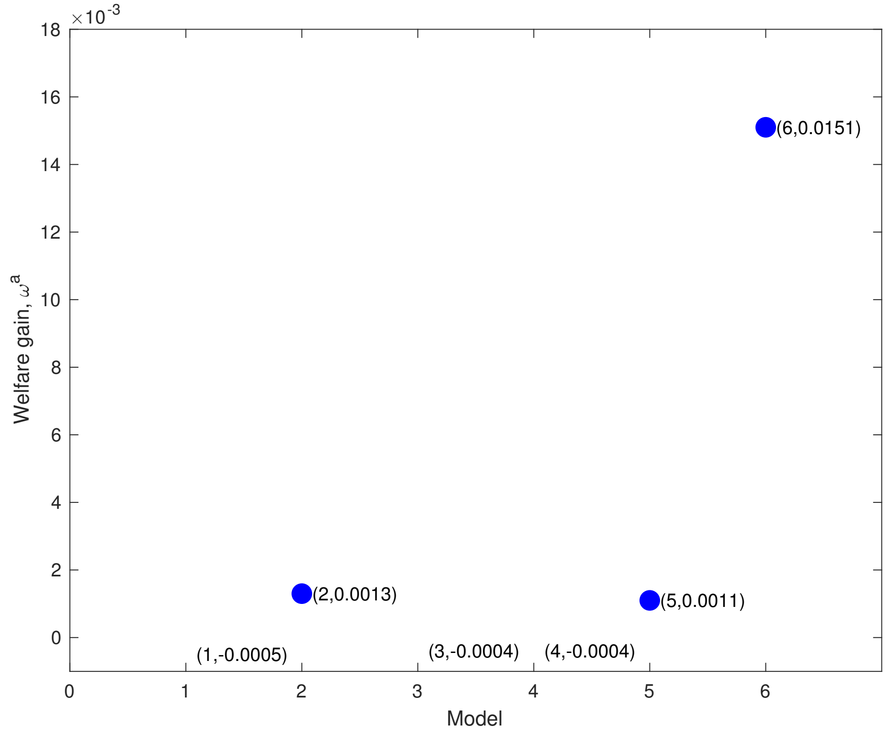
Denote $G_0^b = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}$. The conditional welfare gain (ω^a) is then given by equation (33)

$$\omega^a = (\frac{V_0^a - G_0^b}{V_0^b - G_0^b})^{\frac{1}{1-\epsilon_c}} - 1 \tag{33}$$

Note that, for ease of exposition, we redefine the welfare gain as a positive number if an alternative policy yields a higher level of welfare than the baseline policy. The derivation of the measure of the conditional welfare gain is shown in Appendix A.4.

Figure 2 shows the welfare gain under different models. There are two main points to be taken from the results. First, the following changes in policy parameters increase welfare: an increase in the transaction efficiency of CBDC, a decrease in the tax rate, and an increase in the nominal rate of return on CBDC. Second, it is possible to rank these three changes as follows based on the relative welfare gains from implementing them: increase in the nominal rate of return on CBDC $\succ$ increase in the transaction efficiency of CBDC $\succ$ decrease in the tax rate. The following changes in policy parameters marginally reduce welfare: a decline in the transaction efficiency of cash, an increase in the probability of detection of tax evasion, and an increase in the penalty for evading taxes.

Figure 2: Welfare gain in alternative models relative to baseline model



Notes: This figure shows that there are welfare gains in alternative model 2 (increase in the transaction efficiency of CBDC), alternative model 5 (reduction in tax rate applied to transactions using CBDC), and alternative model 6 (higher rate of return on CBDC) relative to the baseline model.

3.5. Negative nominal rate of return on CBDC

One of the key attractions of a CBDC is that it allows the central bank to impose a negative nominal interest rate even on outside money, which is not possible with cash. This would allow a central bank facing deflationary risks to circumvent the constraints on conventional monetary policy implied by the zero lower bound. In our model, such a policy would not necessarily cause CBDC demand to collapse to zero because CBDC still has a number of advantages that allow it to retain value as a medium of exchange. That is, even when the CBDC's nominal rate of return is dominated by those of other assets available to the representative agent, the model still yields steady states with positive CBDC holdings (our preliminary analysis shows that there are no steady states when the return on CBDC becomes excessively negative; in future work, we intend to more carefully pursue a characterization of this threshold).

A change in the rate of return on CBDC, which is set by the government/central bank, will of course affect the entire structure of rates of return on the available menu of assets as well as their shares of total assets. Table 5 lists the different definitions and representations of rates of return for various assets.

Table 5: Formulas for rates of return (in percent)

Description	**Shorthand**	**Capital**	**Bond**	**CBDC**	**Cash**
Gross nominal rate of return	GNRR	$f_k(\bar{k})(1+\bar{\pi})$	$\bar{I}^b$	$\bar{I}^d$	$\bar{I}^c$
Gross real rate of return	GRRR	$f_k(\bar{k})$	$\frac{\bar{I}^b}{1+\bar{\pi}}$	$\frac{\bar{I}^d}{1+\bar{\pi}}$	$\frac{\bar{I}^c}{1+\bar{\pi}}$

Notes: This table shows the definitions of different rates of return for various assets.

As an illustrative exercise, we analyze the steady state values of the model when the central bank sets the gross nominal rate of return on CBDC at 0.95 percent (this corresponds to a 10 percent decrease in the gross nominal rate of return on CBDC relative to the baseline model). The results in the block AM in Table 6 show that the share of CBDC falls by about 20 percentage points (from 39.28 to 19.02). The shares of cash and capital decrease while the share of bonds increases. This can be seen more clearly in the last block of the table, which shows the percentage changes in asset shares relative to the baseline model. When the gross nominal rate of return of CBDC (0.95%) is lower than that of bonds (1.05%), bonds behave as a better storage technology. Hence, agents shift from CBDC towards bond holdings. Cash holdings increase when the nominal rate of return on CBDC is negative. However, since bond holdings increase substantially as they become the dominant store of value, the share of cash holdings falls.[17]

[17]Additionally, private consumption falls by 9 percent. This is because of the decline in CBDC holdings, which is the efficient medium of exchange, and the only modest increase in cash holdings. Agents switch to saving through bonds and reduce their consumption.

Table 6: Steady state implications of negative nominal interest rate on CBDC

Model	**BM**				**AM**				Δ%			
Variable	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**
GNRR	0.15	1.06	1.00	1.05	0.15	1.06	1.00	0.95				
GRRR	0.15	1.05	0.99	1.04	0.15	1.05	0.99	0.94				
Position	3.23	1.45	0.56	3.39	3.23	4.74	0.61	2.01				
Share	37.44	16.80	6.48	39.28	30.48	44.73	5.77	19.02	-6.96	37.93	-0.71	-20.26

Notes: BM refers to the baseline model in which the gross nominal rate of return on CBDC is 1.05 percent. AM is the alternative model with a gross nominal rate of return of 0.95 percent on CBDC. The first two rows show various rates of return, the third row shows asset positions, and the last row shows share of each of the assets in total assets. The Δ% column block shows the percentage change in the shares of various assets between baseline and alternative models.

One question is whether negative and positive changes to the CBDC interest rate have symmetric effects on asset shares. In Table 7, we examine the effects of a 1 percent increase in the gross rate of return on CBDC. This increase in the CBDC interest rates results in an increase in the share of CBDC that is much greater (in proportional terms) than the fall in its share in response to a cut in the CBDC interest rate. Holdings of bonds decline significantly when the CBDC interest rate rises. That is, the asset held primarily as a store of value, rather than as a factor of production or a medium of exchange, is most affected by changes in the CBDC interest rate.

Table 7: Steady state implications of positive nominal interest rate on CBDC

Model	**BM**				**AM**				Δ%			
Variable	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**
GNRR	0.15	1.06	1.00	1.05	0.15	1.06	1.00	1.06				
GRRR	0.15	1.05	0.99	1.04	0.15	1.05	0.99	1.05				
Position	3.23	1.45	0.56	3.39	3.23	0.89	0.55	3.67				
Share	37.44	16.80	6.48	39.28	38.70	10.67	6.60	44.03	1.26	-6.13	0.12	4.75

Notes: BM refers to the baseline model. AM is the alternative model with a gross nominal rate of return of 1.06 percent on CBDC. Rate of return, asset position, and asset shares when the nominal rate of return on CBDC is positive. The Δ% column block of last row represents the percentage change between baseline and alternative models.

4. Extensions

In this section, we consider extensions to the basic model. First, we examine the effects of setting the CBDC interest rate at zero, the same rate of return as cash. Second, we study the transition dynamics of private consumption in response to productivity shocks. Third, we analyze how the government can influence holdings of CBDC relative to other assets via direct helicopter drops that increment the stock of CBDC held by agents. Fourth, we show how to structure the rate of return on CBDC ex-ante to avoid capital flight from bonds into CBDC. Fifth, we investigate the effects of introducing a tax on earnings from bond holdings. Sixth, we show how the government can encourage CBDC holdings by imposing a tax on cash holdings above a predetermined threshold. Finally, we consider some special cases of our more general model.

4.1. Zero net interest rate on CBDC

In our baseline analysis, we have assumed a positive net interest rate on CBDC. Central banks considering the issuance of CBDC seem to view it, at least in the initial stages, as being a cash-like instrument, which would imply a zero interest rate (see BIS, 2020).

Table 8: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	33.27	33.29	33.13	33.25	33.25	33.32	33.93
Bond holdings	34.64	34.59	34.75	34.68	34.68	34.44	32.03
Cash holdings	6.07	6.05	6.03	6.04	6.04	6.07	6.13
CBDC holdings	26.03	26.07	26.09	26.03	26.03	26.17	27.90
CBDC (% change)	-	0.17	0.26	0.03	0.03	0.54	7.21

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1 percentage point increase in $\bar{I}_d$, i.e. $\bar{I}_d^{BM} = 1.00$ and $\bar{I}_d^{AM-VI} = 1.01$ (rate of return on CBDC).

Table 8 shows the counterparts of our main results when the net interest rate on CBDC is set to zero. A few points are worth noting. First, agents still hold CBDC for transaction purposes but the steady state shares are lower than in the baseline results (Table 2). Second, cash and CBDC co-exist in this equilibrium but, since neither of them serves as a store of value, the shares of bond holdings are in general higher as well. Thus, all of our key results are preserved and do not hinge on the assumption of a positive interest rate on CBDC.[18]

4.2. Transition dynamics in response to productivity shocks

Our analysis thus far provides comparisons across steady states. In our model, capital and, hence, output remain the same across different steady states. In order to better understand how economic activity evolves between two steady states due to various shocks, we now conduct a simple exercise to determine the impulse response function for private consumption in response to productivity shocks. This allows us to characterize, in an admittedly crude manner, the transition path between two steady states in response to a particular shock. Let productivity, denoted by A, follow an AR(1) process

$$log(A_t/\bar{A}) = \rho_a log(A_{t-1}/\bar{A}) + e_a$$

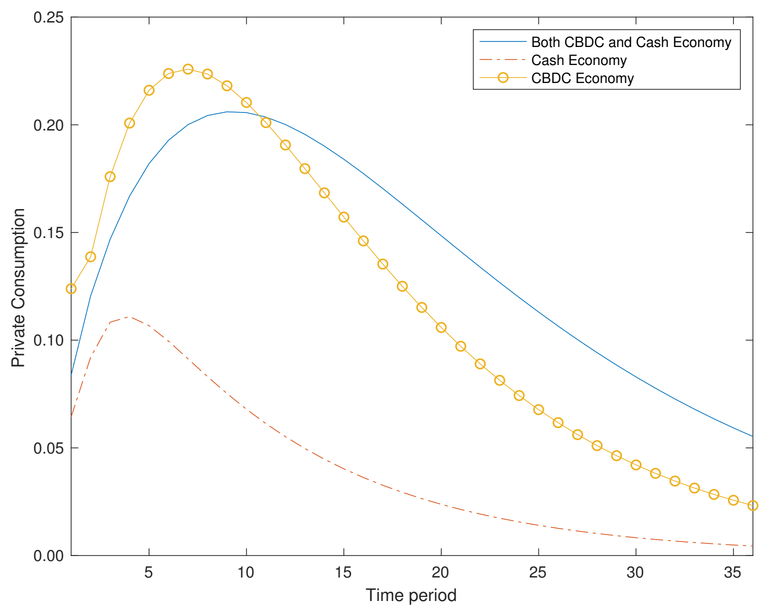
where $0 < \rho_a < 1$ and $e_a \sim N(0, \sigma_a^2)$.

Figure 3 shows the impulse response functions of private consumption in response to a (transitory) one standard deviation positive shock to productivity. The blue solid line depicts the consumption profile for an economy with both cash and CBDC, the red dashed line shows the consumption profile for an economy with cash only, and the yellow bubble line displays the consumption profile for an economy with CBDC only. The positive impact on consumption is largest in the case of an economy with both cash and CBDC. More formal welfare calculations suggest that, relative to an economy with both cash and CBDC, the welfare gain is about 18 percent lower in the case of an economy with only CBDC and 73

[18]The experiments conducted later in this section are also robust to eliminating the positive interest rate on CBDC.

percent lower in an economy with only cash. The smaller welfare gain in the case of a cash economy is probably on account of the CBDC being usable as both a medium of exchange and store of value while cash is less efficient in both those dimensions.

Figure 3: Consumption profiles in different cash-CBDC regimes



Notes: This figure shows the profiles of private consumption under different regimes in response to a one standard deviation shock to productivity.

The response to a negative productivity shock would be symmetric. This sets the stage, in the next sub-section, for illustrating the use of helicopter drops of money as a countercyclical policy tool in an environment with CBDC.

4.3. Helicopter drops

In our model, helicopter drops of money (as a countercyclical policy tool) can be implemented directly by affecting the holdings of CBDC rather than through the conventional view of helicopter drops as fiscal policy measures financed by monetary expansion. Similarly, a negative interest rate on CBDC can take the form of a haircut, in which agents face an effective nominal negative rate of return on CBDC as the balances on their accounts at the central bank shrink. The model in this paper does not incorporate endogenous output responses but we can still use it to show how such policies affect the entire portfolio of assets held by agents and their intertemporal allocation decisions.

Define h as the probability of a recession that triggers a policy response in the form of a helicopter drop of CBDC and s as the size of the helicopter drop. More precisely, h is the probability that an agent gains an s percent increment to her/his CBDC holdings. The central bank implements a helicopter drop of CBDC or sets a negative nominal rate of return on CBDC in bad states of the economy. The parameter h is set based on the frequency of recessions in the past 100 years in the United States, i.e., $h = \frac{\#Recession}{100}$. The central bank sets s and the nominal rate of return on CBDC. Thus, the helicopter drops in this model are ex-ante probabilistic increments to CBDC holdings.

4.3.1. Modification of baseline model

As before, the representative agent maximizes the present discounted value of utility by choosing private consumption goods, public consumption goods, physical capital, bonds, cash, and CBDC holdings. However, the nominal rate of return on CBDC in the RHS of the budget constraint differs from that in the baseline model. The revised budget constraint is as follows:

$$\begin{aligned} c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}(h(1+s)dc_{t-1} + (1-h)dc_{t-1}) + \frac{ca_{t-1}}{1+\pi_t} \end{aligned}$$

All the FOCs remain the same, except with respect to the choice variable dc. The FOC for dc yields

$$
\begin{aligned}
\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} (\frac{I_t^d}{1+\pi_{t+1}}(h(1+s)+(1-h)) \\
& - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}))
\end{aligned} \tag{34}
$$

In the steady state,

$$
1 = \beta(\frac{\bar{I}^d}{1+\bar{\pi}}(h(1+s)+(1-h)) - \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}})) \tag{35}
$$

4.3.2. Comparison of steady state relative to the baseline model

The parameter values in this exercise are the same as in the baseline model, except for the new parameters h and s, which are set as follows:

Table 9: Additional parameters for helicopter drop model

Parameter	Description	Baseline Model	Helicopter drop Model
h	Probability of helicopter drop	-	0.01
s	Percentage of helicopter drop	-	0.15

Notes: These are the new parameters in addition to those listed in Table 1.

The steady state shares of different assets are shown in Table 10. We observe that the steady state share of CBDC increases in comparison with that in the baseline model. The representative agent has an incentive to increase the share of CBDC holdings due to the probability of a helicopter drop increment.

Table 10: Shares in steady state for helicopter drop model

Variable	Description	Baseline	Helicopter drop
$\bar{k}^{share}$	Physical capital	37.44	38.14
$\bar{b}^{share}$	Bond holdings	16.80	15.68
$\bar{ca}^{share}$	Cash holdings	6.48	6.62
$\bar{dc}^{share}$	CBDC holdings	39.28	39.56

Notes: Comparison of asset shares between baseline and helicopter drop models.

4.3.3. Implementing helicopter drops

The probability of a helicopter drop, which depends on the state of the economy, is taken as an exogenous parameter by the central bank. This parameter h, is assumed to take any one of the following values: $h \in \{0.01, 0.05, 0.10, 0.15, 0.20\}$. The central bank then has two instruments under its discretion in implementing helicopter drops. First, it can set different sizes of helicopter drops (increments to the beginning of period CBDC stocks) for a given nominal rate of return. Second, it can set different nominal rates of return on CBDC for a given helicopter drop percentage. We determine the relationship between the helicopter drop percentage (s) and share of CBDC ($\bar{dc}^{share}$) under different states of the economy (captured by h) for a fixed gross nominal rate of return ($\bar{I}^d$). The gross nominal rate of return on CBDC is chosen from among the following values: $\bar{I}_d \in$\{1.04,1.05,1.06\}. The steady state CBDC shares are determined for each of those values.

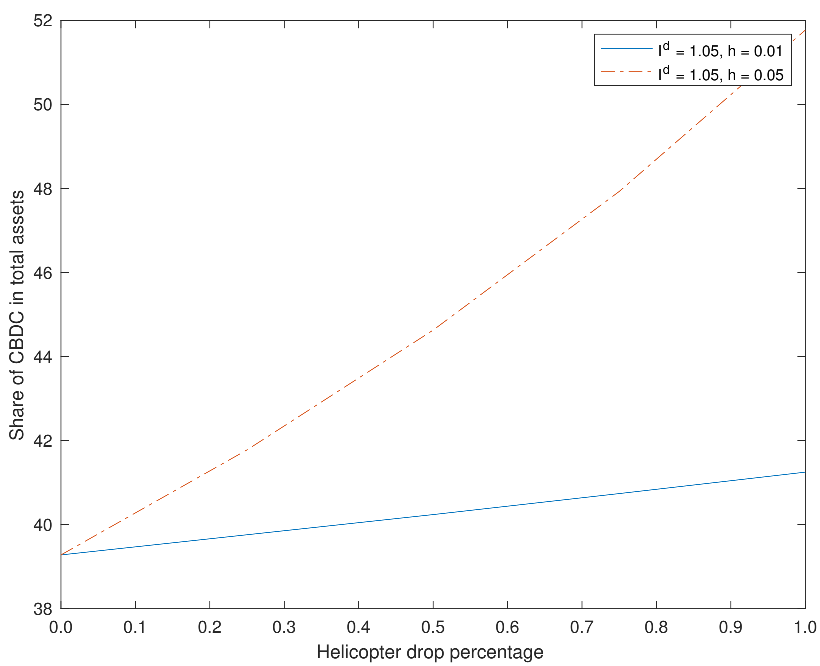
Table 11 and Figure 4 capture the relationship between s and $\bar{dc}^{share}$. While the first column block of Table 11 shows the effect of s on $\bar{dc}^{share}$ at $\bar{I}^d = 1.04$ and $h = 0.01$, the second column block shows the effect of s on $\bar{dc}^{share}$ at $\bar{I}^d = 1.04$ and $h = 0.05$. Similarly, the third and fourth column blocks represent the effects of s on $\bar{dc}^{share}$ for different states of the economy when $\bar{I}^d = 1.05$. For a fixed value of $\bar{I}_d$ and h, the CBDC share increases with the size of the helicopter drop. The rise in the CBDC share is much steeper as the probability of a bad state of the economy rises. Also, for a fixed nominal rate of return on CBDC (I^d) and a given value of the helicopter drop percentage (s), the CBDC share rises with increases in the probability of a helicopter drop (h).

Table 11: Effects of probability and size of helicopter drop on CBDC shares in total asset holdings

$\bar{I}^d = 1.04$ & $h = 0.01$		$\bar{I}^d = 1.04$ & $h = 0.05$		$\bar{I}^d = 1.05$ & $h = 0.01$		$\bar{I}^d = 1.05$ & $h = 0.05$	
s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$
0.00	35.64	0.00	35.64	0.00	39.28	0.00	39.28
0.25	36.02	0.25	37.67	0.25	39.76	0.25	41.78
0.50	36.42	0.50	39.96	0.50	40.24	0.50	44.63
0.75	36.83	0.75	42.57	0.75	40.74	0.75	47.93
1.00	37.24	1.00	45.55	1.00	41.25	1.00	51.77

Notes: CBDC shares for different values of the size of the helicopter drop (increment, in percent, to the beginning of period CBDC stocks). $\bar{I}^d$ denotes the gross nominal rate of return on CBDC, h denotes probability of helicopter drop, s denotes size of helicopter drop, and $\bar{dc}^{share}$ denotes the CBDC share.

Figure 4: Relationship between size of helicopter drop and CBDC share



Notes: This figure shows how the share of CBDC in total assets varies with size of the helicopter drop, with the gross nominal rate of return on CBDC set at 1.05% and the probability of the helicopter drop set at 0.01 (solid blue line) and 0.05 (dotted red line), respectively.

4.4. Limiting flight into CBDC

We now show how the rate of return on CBDC can be designed ex-ante to avoid capital flight into CBDC, which could be triggered by flight to safety during a financial panic or a severe recession.[19] The model in this paper does not feature bank deposits, so the flight would be out of bonds (and, potentially, other assets) but the mechanism we discuss below would be equally relevant for limiting flight from bank deposits into CBDC. Of course, any flight into CBDC for reasons of safety would hinge on the credibility of the central bank issuing the CBDC and also on the fiscal soundness of the country's government, both of which might be tested in periods of financial crisis.

4.4.1. Design

The monetary authority randomly chooses a cut-off CBDC value, dc^c, that pins down the value of CBDC above which agents are required to pay a fee to hold CBDC. Let agents' CBDC holdings at a given point of time be denoted by dc_t. CBDC holdings provide a constant rate of return, as in the baseline model, when dc_t is less than or equal to dc^c. However, agents pay an increasing fee on CBDC holdings when dc_t is greater than dc^c. This fee translates into a decreasing return when dc_t is greater than dc^c. The rate of return on CBDC is designed as follows:

$$I_t^d(dc_t, dc^c) = \begin{cases} \bar{I}^d, & \text{if } dc_t \leq dc^c \\ \bar{I}^d - I^f(dc_t - dc^c), & \text{if } dc_t > dc^c \end{cases}$$

where $I^f(dc_t - dc^c)$ is an increasing function of $dc_t - dc^c$.

Denote $\bar{dc}$ as the steady state value of CBDC holdings. The steady state rate of the return on CBDC is then determined as

[19] An alternative approach, adopted by the Central Bank of the Bahamas in the design of its CBDC called the "sand dollar," is to impose a quantity restriction on the size of retail CBDC accounts. See https://www.centralbankbahamas.com/news/press-releases/project-sand-dollar-the-central-bank-identifies-preferred-technology-solutions-provider-for-bahamas-digital-currency.

$$I_t^d(\bar{dc}, dc^c) = \begin{cases} \bar{I}^d, & \text{if } \bar{dc} \leq dc^c \\ \bar{I}^d - I^f(\bar{dc} - dc^c), & \text{if } \bar{dc} > dc^c \end{cases}$$

4.4.2. Functional form for the penalty function

Assume $I^f(dc_t - dc^c) = \omega_f(dc_t - dc^c)$. That is, the fee rises linearly with the excess of CBDC holdings above the cut-off level.

$$I_t^d(dc_t, dc^c) = \begin{cases} \bar{I}^d, & \text{if } dc_t \leq dc^c \\ \bar{I}^d - \omega_f(dc_t - dc^c), & \text{if } dc_t > dc^c \end{cases}$$

4.4.3. Modification of the basic model

The only modification relative to the baseline model is the revised budget constraint:

$$\begin{aligned} c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \eta g_t - g_t + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \mathbb{1}_{dc_t \leq dc^c} \frac{I_{t-1}^{dl}}{1+\pi_t} dc_{t-1} + \mathbb{1}_{dc_t > dc^c} \frac{I_{t-1}^{du}}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned}$$

where $\mathbb{1}$ is the indicator function, $I_{t-1}^{dl} = \bar{I}^d$, and $I_{t-1}^{du} = \bar{I}^d - \omega_f(dc_{t-1} - dc^c)$. All the FOCs remain the same, except with respect to the choice variable dc. The FOC with respect to CBDC holdings under the transformed budget constraint takes the following form:

$$\begin{aligned} \frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1 + \phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \\ & (\mathbb{1}_{dc_t \leq dc^c} (\frac{I_t^{dl}}{1+\pi_{t+1}} + \frac{dc_{t+1}}{1+\pi_t} * \frac{\partial I_t^{dl}}{\partial dc_t}) \\ & + \mathbb{1}_{dc_t > dc^c} (\frac{I_t^{du}}{1+\pi_{t+1}} + \frac{dc_t}{1+\pi_{t+1}} * \frac{\partial I_t^{du}}{\partial dc_t}) \\ & - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}) \end{aligned} \tag{36}$$

In the steady state,

$$
\begin{aligned}
1 =& \beta(\mathbb{1}_{\bar{dc} \leq dc^c}(\frac{\bar{I}^{dl}}{1+\bar{\pi}} + \frac{\bar{dc}}{1+\bar{\pi}}\frac{\partial \bar{I}^{dl}}{\partial \bar{dc}}) + \mathbb{1}_{\bar{dc} > dc^c}(\frac{\bar{I}^{du}}{1+\bar{\pi}} + \frac{\bar{dc}}{1+\bar{\pi}}\frac{\partial \bar{I}^{du}}{\partial \bar{dc}}) \\
&- \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}})
\end{aligned}
\tag{37}
$$

where $\frac{\partial \bar{I}^{dl}}{\partial \bar{dc}} = 0$ and $\frac{\partial \bar{I}^{du}}{\partial \bar{dc}} = -\omega_f$.

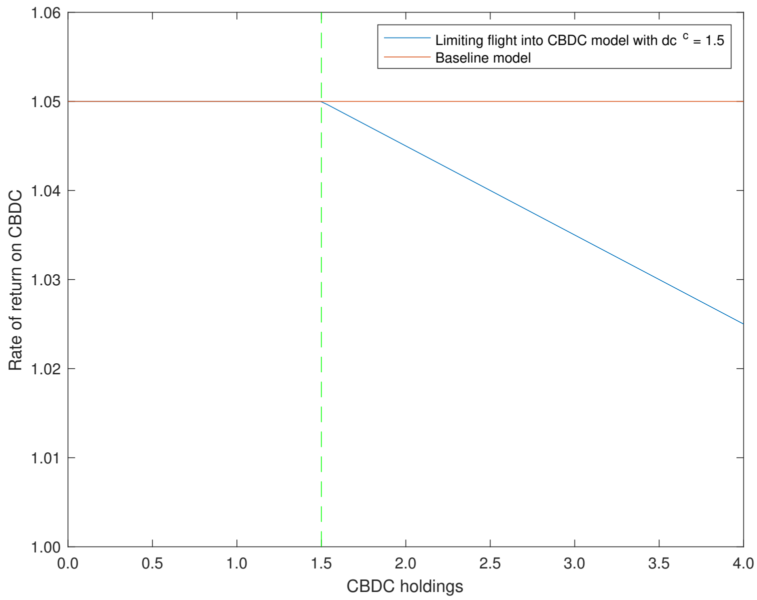
An interesting departure from the earlier setting is that the rate of return on CBDC is now determined endogenously:

$$
\bar{I}^d = \mathbb{1}_{\bar{dc} \leq dc^c}(\bar{I}^d) + \mathbb{1}_{\bar{dc} > dc^c}(\bar{I}^d - \omega_f(\bar{dc} - dc^c)) \tag{38}
$$

4.4.4. Results

We set the penalty fee parameter $\omega_f = 0.01$. That is, the CBDC interest rate decreases by 1 percentage point for a unit increase in CBDC holdings above the cut-off CBDC threshold. Figure 5 shows the relationship between the nominal rate of return of CBDC and CBDC holdings for the baseline model and for the extended model with the cut-off CBDC value set at 1.50. This shows how the monetary authority can design the nominal rate of return on CBDC ex-ante to disincentivize CBDC holdings above the cut-off value.

Figure 5: Relationship between CBDC holdings and rate of return of CBDC



Notes: This figure shows how the rate of return is designed by the central bank ex-ante to limit flight into CBDC. The central bank imposes a fee on CBDC holdings above a given threshold. This fee is linear in the amount by which CBDC holdings exceed the threshold.

The steady state shares of different assets are reported below in Table 12. The steady state share of CBDC in the model with the fee on excess CBDC holdings (Column A) is lower than that in baseline model. In a period of financial distress, agents might wish to move completely to CBDC. This phenomenon is similar to having the CBDC steady state value above the cut-off CBDC. However, the increasing fee (or decreasing return on CBDC) regime kicks in that results in the gross nominal return on CBDC being lower than that of bonds. This prevents bond holdings from collapsing to zero.

Table 12: Steady state shares (in percent) for model with fee on CBDC holdings above threshold

Description	BM	A
Share		
Physical capital	37.44	40.72
Bond holdings	16.80	18.37
Cash holdings	6.48	8.19
CBDC holdings	39.28	32.72
Gross Nominal Rate of Return		
CBDC	1.05	1.04
Bond	1.06	1.06
Positions		
CBDC holdings	3.39	2.59
CBDC cut-off	-	1.50

Notes: BM refers to the baseline model. Column A refers to the alternative model with a fee on CBDC holdings above the threshold, which is set at 1.5. The table shows the steady state shares of different assets in total assets. The last two blocks of the table show the gross nominal rate of return on CBDC, which is endogenous in the alternative model, and the absolute level of CBDC holdings.

4.5. Tax on bond holdings

In our baseline model, the only tax is on CBDC holdings, which captures taxes on consumption transactions using CBDC. In this extension, we modify our baseline model by imposing a 5 percent tax on agents' interest earnings from bond holdings. The non-negativity constraint on bond holdings now becomes pertinent since in this environment a CBDC might, under some settings, be a better store of value than bonds, in addition to being an efficient medium of exchange.

4.5.1. Modification of baseline model

The representative agent's maximization problem now incorporates the tax on bond returns in the RHS of the budget constraint and (as before) a separate non-negativity constraint on bond holdings.

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)]$$

subject to

$$c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1}$$
$$\leq f(k_{t-1}) + (1-\tau_b)\frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$

$$b_t \geq 0$$
$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t}$$

Table 13 shows the steady state shares of different assets under various taxation regimes. Setting a positive tax rate on bond yields has no impact on the shares of bonds in total assets (Column A). Rather there is an increase in the gross nominal rate of return on bonds, which offsets the positive tax rate on bond yields.[20] When the nominal rate of interest on CBDC is increased such that, after accounting for the tax on bond returns, the rate of return on CBDC approaches (but is still below) that of bonds, bond holdings hit the boundary condition (Column B). Thus, the increase in the CBDC interest rate and the tax on bond holdings reinforce each other in making CBDC more attractive both as a store of value and medium of exchange.

[20] The after-tax gross nominal rate of return on bonds for column A is $(1-\tau_b)I^b = (1-0.05)1.12 = 1.06$. Thus, the after-tax gross nominal rate of return on bonds in column A is equal to the gross nominal rate of return on bonds in baseline model (which has no taxation on interest income from bond holdings).

Table 13: Steady state shares (in percent) for model with tax on bond yields and non-negative bond holdings

Description	BM	A	B
Share			
Physical capital	37.44	37.44	12.36
Bond holdings	16.80	16.80	0.00
Cash holdings	6.48	6.48	2.78
CBDC holdings	39.28	39.28	84.86
Gross Nominal Rate of Return			
CBDC	1.05	1.05	1.09
Bond	1.06	1.12	-
Tax rate			
CBDC	0.08	0.08	0.08
Bond	0.00	0.05	0.05

Notes: BM refers to the baseline model. The next two columns show shares of various assets when a tax is levied on interest earnings from bonds (Column A) and, in addition, a non negativity restriction is imposed on bond holdings in conjunction with an increase in the CBDC interest rate (Column B).

4.6. Influencing CBDC holdings by taxing cash

In principle, the government can levy a tax on cash holdings to encourage a greater use of CBDC in consumption and other transactions. For example, in its 2019 budget session, the Indian government considered the introduction of a tax of 2% on cash withdrawals of more than 10 million Indian rupees (INR; roughly $135,000 at the November 2020 exchange rate) in a year. In 2005, the government had introduced a Banking Cash Transaction Tax (BCTT) that imposed a tax of 0.1% on cash withdrawals exceeding 25 thousand INR in a single day.[21] In 2019, the Greek government mandated that 30 percent of an individual's

[21] See https://www.financialexpress.com/budget/budget-2019-govt-looks-to-drive-digital-payments-with-2-tax-on-cash-withdrawals-above-rs-1-crore/1633224/;https://economictimes.indiatimes.com/wealth/personal-finance-news/budget-2019-will-the-banking-cash-transaction-tax-make-a-comeback/articleshow/69929250.cms?

income be allocated to digital spending, with a tax imposed on any shortfall from the target. This is an alternative mechanism for penalizing the use of cash in the purchase of goods and services.[22]

We now examine how a tax on cash holdings, if that were feasible to implement, could be used to affect CBDC holdings. Let h_c be the probability that an agent has cash holdings that surpass a threshold value, ca^c. Let τ_c be the tax that is imposed when an agent's cash holdings exceed this threshold. The probability of the imposition of such a tax on cash holdings and the structure of the tax are designed as follows:

$$Probability = \begin{cases} h_c, & \text{if } ca_t > ca^c \\ 1 - h_c, & \text{if } ca_t \leq ca^c \end{cases}$$

$$\tau_c(ca_t) = \begin{cases} \tau_c, & \text{w.p. } h_c \\ 0, & \text{w.p. } (1 - h_c) \end{cases}$$

This results in the following revised budget constraint:

$$c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t$$
$$+h_c\tau_c ca_t + (1-h_c)0ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$

All the FOCs remain the same, except with respect to the choice variable ca. The FOC for ca yields

$$\begin{aligned} \frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})}(1+h_c\tau_c) =& \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \\ & (\frac{1}{1+\pi_{t+1}} - \frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} - \frac{\phi_{ca}(c_{t+1}, ca_t, dc_t)}{1+\pi_{t+1}}) \end{aligned} \tag{39}$$

In the steady state,

$$1 + h_c\tau_c = \beta(\frac{1}{1+\bar{\pi}} - \frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}} - \frac{\phi_{ca}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \tag{40}$$

[22]See https://www.dontmesswithtaxes.com/2019/12/greece-to-force-more-digital-transactions-new-tax-penalty-to-thwart-tax-cheats.html

4.6.1. Comparison of steady state relative to the baseline model

The parameter values are the same as in the baseline model, except for the new parameters h_c and τ_c, which are chosen as follows:

Table 14: Additional parameters for model with tax on cash holdings

Parameter	Description	Baseline Model	Tax on Cash Model
h_c	Probability of cash holding above threshold	-	0.05
τ_c	Tax-rate on cash holding above threshold	-	0.08

Notes: These are the new parameters in addition to those listed in Table 1.

The steady state values of different assets are reported in Table 15. The steady state value of CBDC is higher than in the baseline model. Taxing cash holdings causes agents to use relatively more CBDC than cash to purchase private consumption goods, which results in an increase in the CBDC position. The tax on cash (along with the existing tax on CBDC) also leads to a shift away from consumption toward higher savings in the form of bonds.

Table 15: Positions of various assets in model with tax on cash

Variable	Description	Baseline	Tax on Cash	Percent change
$\bar{k}$	Physical capital	3.2274	3.2274	0.00
$\bar{b}$	Bond holdings	1.4483	1.4822	2.34
$\bar{ca}$	Cash holdings	0.5589	0.5565	-0.42
$\bar{dc}$	CBDC holdings	3.3866	3.3918	0.15

Notes: Comparison of asset positions in the model with a tax on cash holdings relative to the baseline model.

4.6.2. Effects of probability of cash holdings above threshold level

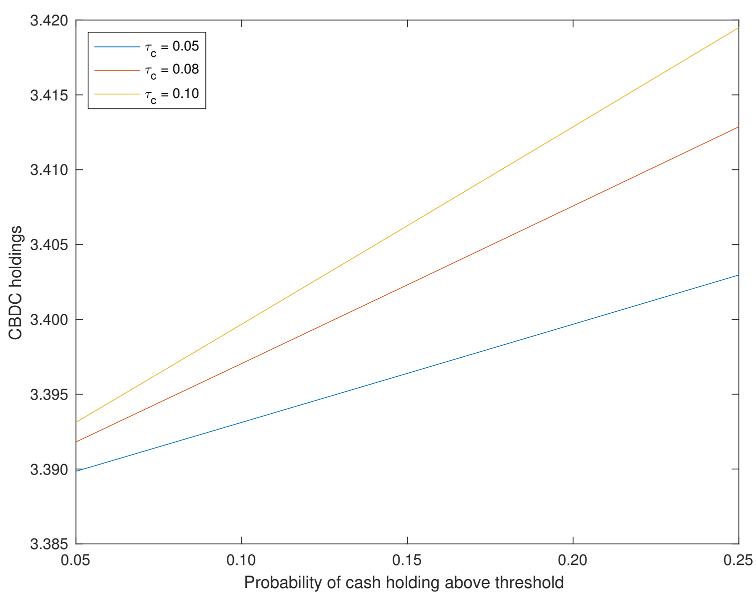
For this experiment, we keep the tax rate on cash holdings constant at one of the following values: $\tau_c \in \{0.05, 0.08, 0.10\}$. For a given value of τ_c, CBDC holdings in the steady state are determined for different values of the probability of cash holdings being above the threshold value. Table 16 and Figure 6 capture the relationship between τ_c and $\bar{dc}$. For a given value of τ_c, CBDC holdings rise with the increase in the probability of cash holdings above the threshold value. Also, for a given value of h_c, CBDC holdings rise when there is an increase in the tax rate on cash holdings. This shows that a higher level of cash holdings and/or a higher tax rate on cash holdings incentivizes agents to increase holdings of CBDC for transaction purposes.

Table 16: Effects of cash holdings above threshold level on CBDC holdings

Variable	τ_c=**0.05**					τ_c=**0.08**					τ_c=**0.10**				
h_c	0.05	0.10	0.15	0.20	0.25	0.05	0.10	0.15	0.20	0.25	0.05	0.10	0.15	0.20	0.25
$\bar{d}c$	3.390	3.393	3.396	3.400	3.403	3.392	3.397	3.402	3.407	3.413	3.393	3.400	3.406	3.413	3.420

Notes: Steady state values of CBDC for different values of the probability of cash holdings being above threshold at which tax on cash kicks in.

Figure 6: Relationship between probability of cash holding above threshold and CBDC holdings



Notes: This figure shows how CBDC holdings vary with the probability of cash holdings being above the threshold level, with the tax rates on cash holdings above the threshold set at 5% (blue line), 8% (red line), and 10% (yellow line), respectively.

4.7. Model variations

In this sub-section, we present results from one additional sensitivity test and two variants of the model that could be considered special cases of the more general baseline model presented in Section 2.

4.7.1. Level parameter of the transaction function

The transaction function includes a level parameter, θ_1, which captures the standard of the payment infrastructure for purchasing goods and services and could be interpreted as a crude measure of the degree of households' access to the payment system. Currently, the value of θ_1 is set to 2 in all the analysis. We now change this parameter in a systematic manner over the range shown in Table 17, holding all other parameters (including policy parameters constant.

Table 17: Level parameter of the transaction function

Parameter	**AM-I**	**AM-II**	**AM-III**	**BM**	**AM-IV**	**AM-V**	**AM-VI**
θ_1	0.5	1	1.5	2	2.5	3	3.5
$\%change$	-75%	-50%	-25%	0%	25%	50%	75%

Notes: θ_1 varies from -75% to 75% of the original value used in the baseline model.

Table 18 shows the steady state shares of different assets for various settings of θ_1. The share of CBDC increases and that of cash decreases when θ_1 is higher. When the costs of transactions using both cash and CBDC rise (higher value of θ_1), bonds become more preferred assets while the share of physical capital declines. Thus, the overall efficiency of the retail payment infrastructure (which could be taken as a crude proxy for financial inclusion) affects holdings of all types of financial and physical assets.

Table 18: Steady state shares (in percent)

Asset	$\theta_1 = 0.5$	$\theta_1 = 1$	$\theta_1 = 1.5$	$\theta_1 = 2$	$\theta_1 = 2.5$	$\theta_1 = 3$	$\theta_1 = 3.5$
Physical capital	55.27	45.45	40.56	**37.44**	35.20	33.49	32.12
Bond holdings	0.51	9.43	13.92	**16.80**	18.87	20.47	21.75
Cash holdings	7.97	7.21	6.78	**6.48**	6.26	6.09	5.94
CBDC holdings	36.25	37.91	38.75	**39.28**	39.66	39.96	40.19

Notes: Steady state share of various assets are compared across different values of θ_1, the level parameter in the transaction cost function, holding all other parameters (including policy parameters) at their respective baseline levels. The column in bold corresponds to the baseline model in Table 2.

4.7.2. Model variant excluding public goods and government expenditure

We now turn to a special case of our general model by excluding public goods (c_g) and government expenditure (g) to examine if that influences the results. The utility function and the budget constraints of the representative agent and the government are then modified as follows:

Representative Agent

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{41}$$

subject to

$$\begin{aligned} c_t + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I^b_{t-1}}{1+\pi_t} b_{t-1} + \frac{I^d_{t-1}}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{42}$$

$$k_t - (1-\delta)k_{t-1} \leq i_t \tag{43}$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{44}$$

Government

$$\begin{aligned} \frac{I^b_{t-1}}{1+\pi_t} b_{t-1} + \frac{I^d_{t-1}}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \\ = b_t + dc_t + ca_t + \frac{\tau}{1+\pi_t} dc_{t-1} + m\psi(\frac{ca_{t-1}}{1+\pi_t}) \end{aligned} \tag{45}$$

The steady state shares in this more restrictive model show one important difference relative to the baseline model, which is that the share of bond holdings is comparable to that of CBDC holdings. This is because, without any expenditure on public goods, the government pays a higher interest rate on bonds in the steady state. The effects of changes in policy parameters around the new baseline are, however, quite similar to those presented in Table 2.

Table 19: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	19.21	19.22	19.13	19.20	19.20	19.25	19.79
Bond holdings	39.78	39.72	39.77	39.79	39.79	39.43	34.79
Cash holdings	4.16	4.15	4.14	4.15	4.15	4.17	4.23
CBDC holdings	36.85	36.91	36.95	36.86	36.86	37.15	41.19
CBDC (% change)	-	0.17	0.29	0.03	0.03	0.81	11.78

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

4.7.3. Endowment economy with no physical capital

In this variant of our baseline model, we drop physical capital and investment, switching instead to an endowment economy. The representative agent is bestowed with an exogenous, deterministic stream of income (y_t) every period. The maximization problem then simplifies to:

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)] \tag{46}$$

subject to

$$c_t + c_t^g + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \leq y_t + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \tag{47}$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{48}$$

The steady state value of income ($\bar{y}$) is set at 1.15. To enable comparisons with the baseline (more general) model, this value is set such that $f(\bar{k}) - \delta\bar{k} = \bar{y}$, where $\bar{k}$ is the steady state value of capital obtained from the baseline model with capital.

The results indicate that the ordering of the relative shares of bonds, cash, and CBDC remains the same as in the baseline model with capital. Moreover, the effects of changing various policy parameters are qualitatively similar to those in Table 2.

Table 20: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Bond holdings	26.89	26.79	26.96	26.94	26.94	26.26	17.46
Cash holdings	10.37	10.35	10.29	10.33	10.33	10.40	10.78
CBDC holdings	62.73	62.86	62.74	62.73	62.73	63.34	71.76
CBDC (% change)	-	0.13	0.01	0.00	0.00	0.61	9.03

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

5. Conclusion

In this paper, we have developed a simple general equilibrium model to capture the trade-offs between cash and CBDC. The key differences between these two forms of central bank-issued (outside) money include transaction costs (lower for CBDC), possibilities for tax evasion (higher for cash, but with a positive probability of being caught and penalized), and nominal rates of return (zero for cash; potentially positive or negative for CBDC). We showed how different combinations of government policies—such as the level of taxes, the penalty for being caught undertaking tax evasion, and the rate of return on CBDC—can influence the relative holdings of cash and CBDC. We also examined other policies—such as helicopter drops and taxes imposed on cash holdings beyond a predetermined threshold—that can influence the shares of CBDC in total assets (which include CBDC, cash, government bonds, and physical capital). The model provides a framework that can be expanded to incorporate further attributes of various types of official and nonofficial currencies.

To highlight the trade-offs between cash and CBDC under different policy environments, we have abstracted from a number of other considerations. One important aspect that is worth exploring further is the potential role for a CBDC in broadening financial inclusion, although this would require a heterogeneous agent model in order to capture differential levels of access to digital payments. We do not explicitly model one important advantage of cash, which is that it provides anonymity in financial transactions unlike most digital forms of payment. Incorporating this explicitly (rather than viewing our general formulation as capturing various benefits of using cash) would require assigning a utility value for privacy, which could be an interesting exercise in itself.

In addition, we have posited an exogenous growth rate of central bank money (cash plus CBDC) in order to pin down the inflation rate. Making monetary policy endogenous and explicitly modeling the transmission of monetary policy to economic activity would be useful extensions. In the present formulation of the model, activity is determined by supply-side fundamentals since our main interest is in characterizing policies that affect the distribution of asset holdings. Another exercise would be to examine if the share of CBDC in total assets is affected by the uncertainty of returns on CBDC (in contrast to cash, which has a fixed zero nominal rate of return). We leave these and other extensions for future work.

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A. Online Appendix

A.1. Real budget constraint and real liquidity constraint

The nominal budget constraint is as follows:

$$\begin{aligned} P_t c_t + P_t c_t^g + P_t i_t + B_t + P_t \phi(c_t, \frac{CA_{t-1}}{P_t}, \frac{DC_{t-1}}{P_t}) + P_t m \psi(\frac{CA_{t-1}}{P_t}) + CA_t \\ + DC_t + \tau DC_{t-1} \leq P_t f(k_{t-1}) + I_{t-1}^b B_{t-1} + I_{t-1}^d DC_{t-1} + CA_{t-1} \end{aligned} \tag{A.1.1}$$

Divide P_t throughout equation A.1.1

$$\begin{aligned} c_t + c_t^g + i_t + \frac{B_t}{P_t} + \phi(c_t, \frac{CA_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}, \frac{DC_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}) + m\psi(\frac{CA_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}) + \frac{CA_t}{P_t} \\ + \frac{DC_t}{P_t} + \tau \frac{DC_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \leq f(k_{t-1}) + I_{t-1}^b \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + I_{t-1}^d \frac{DC_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{CA_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \end{aligned}$$

Thus, the real budget constraint is as follows:

$$\begin{aligned} c_t + c_t^g + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{A.1.2}$$

The nominal liquidity constraint is as follows:

$$P_t c_t \leq CA_{t-1} + DC_{t-1} \tag{A.1.3}$$

Divide P_t throughout equation A.1.3

$$P_t c_t \leq \frac{CA_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{DC_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}$$

Thus, the real liquidity constraint is as follows:

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{A.1.4}$$

A.2. Dynamic optimization of representative agent problem

The Bellman equation for the optimization problem is as follows:

$$\begin{aligned} V(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \max\{u(c_t) + v(c_t^g) + \beta V(k_t, b_t, ca_t, dc_t) + \lambda_t(\frac{ca_{t-1}}{1+\pi_t} \\ + \frac{dc_{t-1}}{1+\pi_t} - c_t)\} \end{aligned} \tag{A.2.1}$$

s.t.

$$\begin{aligned} c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) \\ + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{A.2.2}$$

The first order conditions with respect to the choice variables yields

$$u_c(c_t) = \beta V_k(k_t, b_t, ca_t, dc_t)(1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})) - \lambda_t \tag{A.2.3}$$

$$v_{c^g}(c_t^g) = \beta V_k(k_t, b_t, ca_t, dc_t) \tag{A.2.4}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_b(k_t, b_t, ca_t, dc_t) \tag{A.2.5}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_{ca}(k_t, b_t, ca_t, dc_t) \tag{A.2.6}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_{dc}(k_t, b_t, ca_t, dc_t) \tag{A.2.7}$$

The envelope conditions with respect to the state variables yields

$$V_k(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)(f_k(k_{t-1}) + 1 - \delta) \tag{A.2.8}$$

$$V_b(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)\frac{I_{t-1}^b}{1+\pi_t} \tag{A.2.9}$$

$$\begin{aligned} V_{ca}(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)(\frac{1}{1+\pi_t} - \frac{m\psi_{ca}(\frac{ca_{t-1}}{1+\pi_t})}{1+\pi_t} \\ - \frac{\phi_{ca}(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})}{1+\pi_t}) + \frac{\lambda_t}{1+\pi_t} \end{aligned} \tag{A.2.10}$$

$$\begin{aligned} V_{dc}(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)(\frac{I_{t-1}^d}{1+\pi_t} - \frac{\tau}{1+\pi_t} \\ - \frac{\phi_{dc}(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})}{1+\pi_t}) + \frac{\lambda_t}{1+\pi_t} \end{aligned} \tag{A.2.11}$$

Updating the envelope conditions by one time period, multiplying by β, and using the first order conditions yields the intertemporal conditions.[23]

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = v_{c^g}(c_t^g) \tag{A.2.12}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(f_k(k_t)+1-\delta) \tag{A.2.13}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}\frac{I_t^b}{1+\pi_{t+1}} \tag{A.2.14}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(&\frac{1}{1+\pi_{t+1}} - \frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} \\ &-\frac{\phi_{ca}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{A.2.15}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(&\frac{I_t^d}{1+\pi_{t+1}} - \frac{\tau}{1+\pi_{t+1}} \\ &-\frac{\phi_{dc}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{A.2.16}$$

A.3. Steady state analysis

The utility function is assumed to be

$$u(c) = \frac{c^{1-\epsilon_c}}{1-\epsilon_c} \tag{A.3.1}$$

$$u_c(c) = c^{-\epsilon_c} > 0 \tag{A.3.2}$$

$$u_{cc}(c) = -\epsilon_c c^{-\epsilon_c-1} < 0 \tag{A.3.3}$$

$$\lim_{c\to 0} u_c(c) = \lim_{c\to 0} c^{-\epsilon_c} = \infty; \lim_{c\to\infty} u_c(c) = \lim_{c\to\infty} c^{-\epsilon_c} = 0 \tag{A.3.4}$$

$$v(c^g) = \frac{(c^g)^{1-\epsilon_g}}{1-\epsilon_g} \tag{A.3.5}$$

$$v_{c^g}(c^g) = (c^g)^{-\epsilon_g} > 0 \tag{A.3.6}$$

$$v_{c^g c^g}(c^g) = -\epsilon_g (c^g)^{-\epsilon_g-1} < 0 \tag{A.3.7}$$

[23]Assuming that the liquidity constraint does not hold in the equilibrium, the Lagrange multiplier associated with that constraint (λ_t) is zero.

$$\lim_{c^g \to 0} v_{c^g}(c^g) = \lim_{c^g \to 0} (c^g)^{-\epsilon_g} = \infty; \lim_{c^g \to \infty} v_{c^g}(c^g) = \lim_{c^g \to \infty} (c^g)^{-\epsilon_g} = 0 \tag{A.3.8}$$

Utility functions are strictly increasing, strictly concave, and satisfy the Inada conditions, which is confirmed by equations (A.3.2)-(A.3.4) and (A.3.6)-(A.3.8).

The production function is given by

$$f(k) = Ak^{\alpha} \tag{A.3.9}$$

where $0 < \alpha < 1$

$$f_k(k) = A\alpha k^{\alpha-1} > 0 \tag{A.3.10}$$

$$f_{kk}(k) = A\alpha(\alpha-1)k^{\alpha-2} < 0 \tag{A.3.11}$$

$$\lim_{k \to 0} f_k(k) = \lim_{k \to 0} \alpha k^{\alpha-1} = \infty; \lim_{k \to \infty} f_k(k) = \lim_{k \to \infty} \alpha k^{\alpha-1} = 0 \tag{A.3.12}$$

The production function is strictly increasing, strictly concave, and satisfies the Inada conditions, which is confirmed by equations (A.3.10), (A.3.11), and (A.3.12).

The penalty function takes the form

$$\psi(ca) = \theta_2 ca^{\nu} \tag{A.3.13}$$

where $\theta_2 > 1$ and $\nu > 1$

$$\psi_{ca}(ca) = \nu\theta_2 ca^{\nu-1} > 0 \tag{A.3.14}$$

The penalty function is strictly increasing, which is guaranteed by equation (A.3.14).

The transaction function is assumed to be

$$\phi(c, ca, dc) = \theta_1 \frac{c^a}{(ca)^{\gamma}(dc)^{\rho}} \tag{A.3.15}$$

where $a > 1$ and $\gamma < \rho$

$$\phi(0, ca, dc) = \theta_1 \frac{0^a}{(ca)^{\gamma}(dc)^{\rho}} = 0 \tag{A.3.16}$$

$$\phi_c(c, ca, dc) = a\theta_1 \frac{c^a}{(ca)^{\gamma}(dc)^{\rho}} \frac{1}{c} > 0 \tag{A.3.17}$$

$$\phi_{ca}(c, ca, dc) = -\gamma\theta_1 \frac{c^a}{(ca)^{\gamma}(dc)^{\rho}} \frac{1}{(ca)} < 0 \tag{A.3.18}$$

$$\phi_{dc}(c, ca, dc) = -\rho\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)} < 0 \tag{A.3.19}$$

$$\phi_{cc}(c, ca, dc) = a(a-1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{c^2} > 0 \tag{A.3.20}$$

$$\phi_{caca}(c, ca, dc) = \gamma(\gamma+1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(ca)^2} > 0 \tag{A.3.21}$$

$$\phi_{dcdc}(c, ca, dc) = \rho(\rho+1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)^2} > 0 \tag{A.3.22}$$

$$\phi_{cac}(c, ca, dc) = -a\gamma\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(ca)c} < 0 \tag{A.3.23}$$

$$\phi_{dcc}(c, ca, dc) = -a\rho\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)c} < 0 \tag{A.3.24}$$

The transaction function satisfies all the conditions mentioned in section 2.1.

Solving for equilibrium in steady state yields the following.

$$\begin{aligned} 1 &= \beta(f_k(\bar{k}) + 1 - \delta) = \beta(A\alpha\bar{k}^{\alpha-1} + (1-\delta)) \\ \Rightarrow \bar{k} &= \left(\frac{A\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} = \Theta_k(A, \alpha, \beta, \delta) \end{aligned} \tag{A.3.25}$$

$$\frac{\bar{I}^b}{1+\bar{\pi}} = \frac{1}{\beta} \Rightarrow \bar{I}^b = \Theta_{I^b}(\beta, \bar{\pi}) \tag{A.3.26}$$

$$\begin{aligned} \bar{c} + \bar{c}^g + \delta\bar{k} + \bar{b} + \phi(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}}) + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) + \bar{ca} + \bar{dc} + \frac{\tau}{1+\bar{\pi}}\bar{dc} \\ = f(\bar{k}) + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} \end{aligned} \tag{A.3.27}$$

$$\bar{c}^g = \eta\bar{g} \tag{A.3.28}$$

$$\bar{g} - \eta\bar{g} + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} = \bar{b} + \bar{dc} + \bar{ca} + \frac{\tau}{1+\pi_t}\bar{dc} + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) \tag{A.3.29}$$

Substituting equations (A.3.28) and (A.3.29) in equation (A.3.27)

$$\bar{c} + \bar{g} + \phi(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}}) = f(\bar{k}) - \delta\bar{k} = A\bar{k}^\alpha - \delta\bar{k} = \theta_3(A, \alpha, \beta, \delta) \tag{A.3.30}$$

$$\frac{(\bar{c})^{-\epsilon_c}}{1 + \phi_c(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})} = (\eta\bar{g})^{-\epsilon_g} \tag{A.3.31}$$

$$
\begin{aligned}
0 = \Delta(\bar{ca} + \bar{dc}) &= \frac{\mu - \bar{\pi}}{1+\bar{\pi}}(\bar{ca} + \bar{dc}) \\
&\Rightarrow \bar{\pi} = \mu = \Theta_{\pi}(\mu)
\end{aligned} \tag{A.3.32}
$$

$$
\begin{aligned}
1 &= \beta\left(\frac{1}{1+\bar{\pi}} - \frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}} - \frac{\phi_{ca}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}\right) \\
\Rightarrow 1 &= \beta\left(\frac{1}{1+\mu} - \frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu} + \gamma\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}} \frac{1}{(\bar{ca})}\right) \\
\Rightarrow 1 - \beta\frac{1}{1+\mu} + \beta\frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu} &= \beta\gamma\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}} \frac{1}{(\bar{ca})}
\end{aligned} \tag{A.3.33}
$$

The government sets the nominal interest rate of CBDC in steady state.

$$
\begin{aligned}
1 &= \beta\left(\frac{\bar{I}^d}{1+\bar{\pi}} - \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \bar{ca}, \bar{dc})}{1+\bar{\pi}}\right) \\
\Rightarrow 1 &= \beta\left(\frac{\bar{I}^d}{1+\mu} - \frac{\tau}{1+\mu} + \rho\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}} \frac{1}{(\bar{dc})}\right) \\
\Rightarrow 1 - \beta\frac{\bar{I}^d}{1+\mu} + \beta\frac{\tau}{1+\mu} &= \beta\rho\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}} \frac{1}{(\bar{dc})}
\end{aligned} \tag{A.3.34}
$$

Dividing equation (A.3.33) by equation (A.3.34)

$$
\begin{aligned}
\frac{1 - \beta\frac{1}{1+\mu} + \beta\frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu}}{1 - \beta\frac{\bar{I}^d}{1+\mu} + \beta\frac{\tau}{1+\mu}} &= \frac{\gamma}{\rho}\frac{(\bar{dc})}{(\bar{ca})} \\
\Rightarrow \bar{dc} &= \theta_4(\bar{ca}, \alpha, \beta, \delta, \gamma, \rho, \tau, m, \bar{I}^d, \theta_2, \mu, \nu)
\end{aligned} \tag{A.3.35}
$$

Substituting equation(A.3.35) in equation(A.3.33)

$$
\begin{aligned}
1 - \beta\left(\frac{1}{1+\mu} - \frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu}\right) &= \beta\gamma\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\theta_4}{1+\mu})^{\rho}} \frac{1}{(\bar{ca})} \\
&\Rightarrow \bar{ca} = \theta_5(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I}^d, \theta_1, \theta_2, \mu, \nu)
\end{aligned} \tag{A.3.36}
$$

So,

$$
\Rightarrow \bar{dc} = \theta_6(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I}^d, \theta_1, \theta_2, \mu, \nu) \tag{A.3.37}
$$

Substituting the value of $\bar{ca}$ and $\bar{dc}$ in equation (A.3.31)

$$\begin{aligned}\frac{(\bar{c})^{-\epsilon_c}}{1+\phi_c(\bar{c},\frac{\theta_5}{1+\mu},\frac{\theta_6}{1+\mu})} &= (\eta\bar{g})^{-\epsilon_g} \\ \Rightarrow \bar{g} = \theta_7(\bar{c},a,\alpha,\beta,\delta,a,\gamma,\rho,\tau,m,\bar{I}^d,\theta_1,\theta_2,\mu,\nu,\eta,\epsilon_c,\epsilon_g)\end{aligned} \tag{A.3.38}$$

Finally, the steady state value of private consumption is obtained from equation (A.3.30)

$$\bar{c}+\theta_7(\bar{c},a,\alpha,\beta,\delta,a,\gamma,\rho,\tau,m,\bar{I}^d,\theta_1,\theta_2,\mu,\nu,\eta,\epsilon_c,\epsilon_g)+\phi(\bar{c},\frac{\theta_5}{1+\mu},\frac{\theta_6}{1+\mu}) = \theta_3(A,\alpha,\beta,\delta)$$

$$\bar{c} = \Theta_c(a,A,\alpha,\beta,\delta,a,\gamma,\rho,\tau,m,\bar{I}^d,\theta_1,\theta_2,\mu,\nu,\eta,\epsilon_c,\epsilon_g) \tag{A.3.39}$$

Thus, the steady state values of cash, CBDC, and government expenditure are obtained from equations (A.3.36), (A.3.37), and (A.3.38) and are functions of the model parameters. The steady state level of bond holdings is obtained from the consolidated government budget constraint.

$$\begin{aligned}\bar{g}-\eta\bar{g}+\frac{\bar{I}^b}{1+\bar{\pi}}b+\frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc}+\frac{\bar{ca}}{1+\bar{\pi}} &= \tau\bar{dc}+m\psi(\frac{\bar{ca}}{1+\bar{\pi}})+\bar{b}+\bar{ca}+\bar{dc} \\ \Rightarrow \Theta_g-\eta\Theta_g+\frac{\Theta_{I^b}}{1+\mu}\bar{b}+\frac{\bar{I}^d}{1+\mu}\Theta_{dc}+\frac{\Theta_{ca}}{1+\mu} &= \tau\Theta_{dc}+m\psi(\frac{\Theta_{ca}}{1+\mu})+\bar{b}+\Theta_{ca}+\Theta_{dc}\end{aligned} \tag{A.3.40}$$

A.4. Welfare gain calculation

$$\begin{aligned}V_0^a =& \sum_{t=0}^{\infty}\beta^t(u((1+\omega^a)c_t^b)+v(c_t^{gb})) \\ =& \sum_{t=0}^{\infty}\beta^t(\frac{((1+\omega^a)c_t^b)^{1-\epsilon_c}}{1-\epsilon_c}+\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}) \\ =& (1+\omega^a)^{1-\epsilon_c}\sum_{t=0}^{\infty}\beta^t\frac{(c_t^b)^{1-\epsilon_c}}{1-\epsilon_c}+\sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g} \\ =& (1+\omega^a)^{1-\epsilon_c}(\sum_{t=0}^{\infty}\beta^t(\frac{(c_t^b)^{1-\epsilon_c}}{1-\epsilon_c}+\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g})-(1+\omega^a)^{1-\epsilon_c}\sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}+\sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g} \\ =& (1+\omega^a)^{1-\epsilon_c}V_0^b+(1-(1+\omega^a)^{1-\epsilon_c})G_0^b \\ =& (1+\omega^a)^{1-\epsilon_c}(V_0^b-G_0^b)+G_0^b \\ \Longrightarrow \omega^a =& (\frac{V_0^a-G_0^b}{V_0^b-G_0^b})^{\frac{1}{1-\epsilon_c}}-1\end{aligned}$$

A.5. Sensitivity tests for policy parameter choices around alternative baselines

In the next set of tables, we present sensitivity tests for the main policy parameters but around the alternative baseline scenarios shown in Table 3. The main results are all preserved. For instance, cash and CBDC co-exist under all of the parameter combinations. Similarly, the share of CBDC holdings rises and that of alternative assets falls when the rate of return on CBDC rises.

Table 21: Steady state shares (in percent) for baseline and alternative models

Description	**BM-I**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	37.10	37.13	36.96	37.07	37.07	37.17	38.55
Bond holdings	7.99	7.88	8.02	8.02	8.02	7.59	0.00
Cash holdings	6.90	6.89	6.87	6.88	6.88	6.91	7.05
CBDC holdings	48.01	48.10	48.14	48.02	48.02	48.32	54.40
CBDC (% change)	-	0.09	0.13	0.01	0.01	0.31	6.39

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table 22: Steady state shares (in percent) for baseline and alternative models

Description	**BM-II**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	36.73	36.77	36.60	36.70	36.70	36.78	37.21
Bond holdings	2.84	2.70	2.84	2.88	2.88	2.54	0.00
Cash holdings	6.13	6.12	6.10	6.11	6.11	6.13	6.17
CBDC holdings	54.30	54.42	54.47	54.32	54.32	54.55	56.62
CBDC (% change)	-	0.12	0.15	0.02	0.02	0.25	2.32

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table 23: Steady state shares (in percent) for baseline and alternative models

Description	BM-III	AM-I	AM-II	AM-III	AM-IV	AM-V	AM-VI
Physical capital	32.22	32.25	32.10	32.20	32.20	32.28	33.73
Bond holdings	13.21	13.09	13.22	13.24	13.24	12.89	3.92
Cash holdings	5.73	5.72	5.70	5.71	5.71	5.73	5.88
CBDC holdings	48.84	48.94	48.98	48.85	48.85	49.10	56.47
CBDC (% change)	-	0.10	0.14	0.01	0.01	0.26	7.60

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table 24: Steady state shares (in percent) for baseline and alternative models

Description	BM-IV	AM-I	AM-II	AM-III	AM-IV	AM-V	AM-VI
Physical capital	40.58	40.60	40.41	40.55	40.55	40.65	41.49
Bond holdings	4.96	4.86	5.02	5.00	5.00	4.56	0.00
Cash holdings	5.59	5.57	5.56	5.57	5.57	5.60	5.66
CBDC holdings	48.87	48.96	49.01	48.89	48.89	49.19	52.85
CBDC (% change)	-	0.09	0.13	0.02	0.02	0.32	3.98

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table 25: Steady state shares (in percent) for baseline and alternative models

Description	**BM-V**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	46.61	46.65	46.44	46.58	46.58	46.66	46.69
Bond holdings	0.41	0.30	0.47	0.45	0.45	0.18	0.00
Cash holdings	7.93	7.91	7.89	7.90	7.90	7.93	7.94
CBDC holdings	45.05	45.15	45.20	45.07	45.07	45.23	45.37
CBDC (% change)	-	0.10	0.15	0.02	0.02	0.18	0.32

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).