Appendix I

Competitive Equilibrium with Incomplete Markets

This appendix gives the system of equations (in terms of stationary variables) characterizing the competitive equilibrium under the incomplete market settings.

Demand equation for flexible price sector household

\[ C_i^f = x_{f,t} y_{f,t} - x_{f,t} C^* \]  

(40)

Supply equation of flexible price sector (food sector) firm

\[ x_{f,t} = \phi_n (y_{f,t})^\psi (C_i^f)^\alpha (A_{f,t})^{-\alpha (1+\psi)} \]  

(41)

Demand equation for sticky price sector household

\[ (C_i^r)^{-\alpha} = \beta E_t \left\{ \frac{(C_{i+1}^r)^{-\alpha} R_t}{\Pi_t} \right\} \]  

(42)

Supply equation of sticky price sector (non-food sector) firm

\[ x_{s,t} y_{s,t} = \theta \frac{E_t \left[ \sum_{j=0}^{1} (\alpha \beta)^j Q_{s,t+j} \left( \frac{P_{s,t}}{P_{s,t+j}} \right)^{-\theta} Y_{s,t+j} MC_{s,t+j}^r(z) \right]}{\theta - 1} - \frac{E_t \left[ \sum_{i=0}^{1} (\alpha \beta)^i Q_{s,t+i} \left( \frac{P_{s,t}}{P_{s,t+i}} \right)^{-\theta} Y_{s,t+i} \right]}{\theta - 1} \]  

(43)

Price index in sticky price good sector

\[ 1 = \left[ \alpha (\Pi_{s,1})^{-(1-\theta)} + (1 - \alpha) x_{s,1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \]  

(44)

Real marginal cost in the sticky price sector

\[ MC_i^r = \phi_n (Y_{s,t})^\psi (C_i^r)^\alpha (A_{s,t})^{-\alpha (1+\psi)} \]  

(45)

Market clearing equation for flexible price good
\[ Y_{f,t} = \lambda y_{f,t} = C_{f,t} = \gamma (x_{f,t})^{-\eta} C_t + (1 + \lambda) C^* \] (46)

Market clearing condition for sticky price good

\[ Y_{s,t} = C_{s,t} = (1 - \gamma)(x_{s,t})^{-\eta} C_t \] (47)

Aggregate Price Index

\[ 1 = \left[ \gamma (x_{f,t})^{1-\eta} + (1 - \gamma)(x_{s,t})^{1-\eta} \right]^\frac{1}{\eta} \] (48)

Relation between headline and sticky price index

\[ x_{s,t} = \frac{\prod_{s,t} x_{s,t-1}}{\Pi_t} \] (49)

Aggregation equation

\[ \lambda C^f_t + C^s_t = C_t = Y_t \] (50)
Appendix II

Derivation of Welfare Gains Associated with Different Targeting Rules

Welfare gain is given by

\[ V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U((1 + \omega)C_t^r, N_t^r) \]

\[ V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(1 + \omega)^{1-\alpha} (C_t^r)^{-\alpha}}{1 - \sigma} \right) - E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \phi \frac{(N_t^r)^{\psi}}{1 + \psi} \right\} \]

\[ = (1 + \omega)^{1-\alpha} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^r)^{-\alpha}}{1 - \sigma} \right) - E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \phi \frac{(N_t^r)^{\psi}}{1 + \psi} \right\} \]

\[ V_0 = (1 + \omega)^{1-\alpha} U_0^r - D_0^r \]

where

\[ U_0^r = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^r)^{-\alpha}}{1 - \sigma} \right) \quad \text{and} \quad D_0^r = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \phi \frac{(N_t^r)^{\psi}}{1 + \psi} \right\} \]

Using \( V_0^r = U_0^r - D_0^r \), we can solve for

\[ \omega = \left[ \frac{V_0^r + D_0^r}{V_0^r + D_0^r} \right]^{\frac{1}{1-\alpha}} - 1 \]
Appendix III

Alternative Market Structures

A. Alternative Structure of Complete Markets

We consider a setting in which consumers can write contracts to insure against idiosyncratic income risks but only after being assigned to a particular sector. One could regard this as a complete market setting conditional on worker assignment to sectors, which is determined ex-ante (before insurance contracts are written). In other words, a household cannot insure against cross-sector income risk.

Households in the Flexible Price Sector

A representative household in the flexible price sector maximizes its lifetime utility given by equation (1) subject to the following budget constraint\(^1\)

\[
P_t C^f_t + B^f_t + \frac{\kappa}{2} \left( B^f_t - B^f \right)^2 = W_t N^f_t + R_{t+1} B^f_{t+1} - P_{f,t} C^*
\]

(51)

where \( B^f_t \) is the quantity of one-period nominal risk free discount bonds bought in period \( t \) and maturing in period \( t+1 \). Maximization with respect to \( C^f_t \) yields the Euler equation

\[
(1 + \kappa (B^f_t - B^f))(C^f_t)^{-\sigma} = \beta E_t \left( \frac{C^f_{t+1}}{\Pi_{t+1}} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}}
\]

(52)

Households in the Sticky Price Sector

A representative household in the sticky price sector maximizes its lifetime utility given by equation (1) subject to the following budget constraint\(^2\)

\[
P_t C^s_t + B^s_t + \frac{\kappa}{2} (B^s_t - B^s)^2 = \int_0^1 W_t(z) N^s_t(z) dz + \int_0^1 \Pi_t(z) dz + R_{t+1} B^s_{t+1} - P_{f,t} C^*
\]

(53)

\(^1\) In order to solve the model by linearizing it around the steady state with available techniques we assume that households face a small quadratic adjustment cost, \( \frac{\kappa}{2} \left( B^f_t - B^f \right)^2 \), where \( \kappa \) is a parameter and \( B^f \) is the steady state value of the bond holding.

\(^2\) In order to solve the model by linearizing it around the steady state with available techniques we assume that households face a small quadratic adjustment costs, \( \frac{\kappa}{2} \left( B^s_t - B^s \right) \), where \( \kappa \) is a parameter and \( B^s \) is the steady state value of the bond holding.
where $B_t^r$ represent the quantity of one period nominal riskless discount bond bought in period $t$ and maturing in period $t+1$. Maximization with respect to $C_t^r$ yields the Euler equation

$$\left(1 + \kappa (B_t^r - B^r)\right)(C_t^r)^{-\omega} = \beta E_t \left\{ \left( \frac{C_{t+1}^r}{\Pi_{t+1}} \right)^{-\omega} \frac{R_t}{\Pi_{t+1}} \right\}$$

(54)

Bond markets clear: $\lambda B_t^f + B_t^r = 0$  

(55)

Equations (41), (43)-(50) of Appendix I and (51)-(55) expressed in terms of stationary variables define the system of equations that, combined with the monetary policy rule and exogenous stochastic processes for $A_{t,f}$ and $A_{t,s}$, determine the equilibrium path of the economy under this setting.

**B. General Case with Credit-Constrained Households in Each Sector**

We consider a more general case where households in each sector can be credit constrained. Let $\lambda_1 > 0$ and $\lambda_2 > 0$ be the fractions of households that have access to financial markets in the flexible price sector and in the sticky price sector, respectively. So in this general setting there are four different kinds of agents in the economy based on the sector of economy and access to financial markets. Here again we assume that households with access to financial markets can only insure against income risks *ex post*.

**Households in the Flexible Price Sector**

**Unconstrained Households**

A representative household that has access to financial markets in the flexible price sector maximizes its lifetime utility given by equation (1) subject to the following budget constraint\(^3\)

$$P_t C_t^f + B_t^f + \frac{\kappa}{2} \left( B_t^f - B^f \right) = W_t^f N_{t+1}^f + R_{t+1} B_{t+1}^f - P_{f,t} C^r$$

(56)

Maximization with respect to $C_t^f$ yields the Euler equation

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\(^3\) In order to solve the model by linearizing it around the steady state with available techniques we assume that households face a small quadratic adjustment cost, $\frac{\kappa}{2} \left( B_t^f - B^f \right)$, where $\kappa$ is a parameter and $B^f$ is the steady state value of the bond holding.
\[(1 + \kappa(B^f_i - B^s))(C^f_i)^{-\alpha} = \beta E_t \left\{ (C^f_{i+1})^{-\alpha} \frac{R_t}{\Pi_{i+1}} \right\} \]  

(57)

The labor supply decision of the household is given by

\[
\phi_n \left( \frac{N_{2i}^f}{(C^f_i)^{-\alpha}} \right) = \frac{W^f_i}{P_t} 
\]

(58)

**Constrained Households**

The representative credit-constrained consumer in the flexible price sector maximizes lifetime utility given by equation (1) subject to the following budget constraint

\[ P_t C^f_{2i} = W^f_i N_{2i}^f - P_{f,s} C^* \]

(59)

The labor supply decision of the household is given by

\[
\phi_n \left( \frac{N_{2i}^f}{(C^f_i)^{-\alpha}} \right) = \frac{W^f_i}{P_t} 
\]

(60)

**Households in the Sticky Price Sector**

**Unconstrained Households**

A representative household in the sticky price sector that has access to financial markets maximizes its lifetime utility given by equation (1) subject to the following budget constraint \(^4\)

\[ P_t C^s_{i,t} + B^s_i + \frac{K}{2}(B^s_i - B^s)^2 = \int_0^1 W^s_i(z)N^s_{i,t}(z)dz + \int_0^1 \Pi_i(z)dz + R_{t+1}B^s_{i+1} - P_{f,s} C^* \]

(61)

Maximization with respect to \( C^s_{i,t} \) yields the Euler equation

\[ (1 + \kappa(B^s_i - B^s))(C^s_{i,t})^{-\alpha} = \beta E_t \left\{ (C^s_{i+1})^{-\alpha} \frac{R_t}{\Pi_{i+1}} \right\} \]

(62)

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\(^4\)In order to solve the model with available techniques by linearizing it around the steady state, we assume that households face a small quadratic adjustment costs, \( \frac{K}{2}(B^s_i - B^s) \), where \( K \) is a parameter and \( B^s \) is the steady state value of the bond holding.
The labor supply decision of the household to a firm indexed by $z$ is given by
\[ \phi_n \frac{(N_{1t}^z(z))^\gamma}{(C_{1t}^z)^{-\alpha}} = \frac{W_t^z(z)}{P_t} \] (63)

**Constrained Households**

A representative credit-constrained household in the sticky price sector maximizes its lifetime utility given by equation (1) subject to the following budget constraint
\[ P_t C_{2t} = \int_0^1 W_t^z(z)N_{2t}^z(z)dz + \int_0^1 \Pi_t(z)dz - P_{f,t}C^* \] (64)

Labor supply decision of the household to a firm indexed by $z$ is given by
\[ \phi_n \frac{(N_{2t}^z(z))^\gamma}{(C_{2t}^z)^{-\alpha}} = \frac{W_t^z(z)}{P_t} \] (65)

**Firms**

**Flexible Price Sector**

Firms in the flexible price sector are price taking and therefore the price of the flexible price good is given by equation (17). Combining this with the labor supply decision of households given by equations (58) and (60) and recognizing that $y_{f1,t} = A_{f,t}N_{1t}^t$ and $y_{f2,t} = A_{f,t}N_{2t}^t$, the supply function of firms in flexible price sector are given by
\[ \frac{P_{f,t}}{P_t} = \phi_n \frac{\left(\frac{y_{f1,t}}{A_{f,t}}\right)^\gamma}{\left(A_{f,t}(C_{1t}^t)^{-\alpha}\right)} \] (66)
\[ \frac{P_{f,t}}{P_t} = \phi_n \frac{\left(\frac{y_{f2,t}}{A_{f,t}}\right)^\gamma}{\left(A_{f,t}(C_{2t}^t)^{-\alpha}\right)} \] (67)
Sticky Price Sector

Since firms are symmetric, in equilibrium they will all choose the same price. The marginal costs of firms held by unconstrained and constrained households are therefore given by

\[
\frac{MC_i}{P_i} = MC_i' = \phi_n \left( \frac{y_{s1t}}{A_{s,t}} \right) \phi \left( C_{1t} \right)^{-\sigma}
\]

(68)

\[
\frac{MC_i}{P_i} = MC_i'' = \phi_n \left( \frac{y_{s2t}}{A_{s,t}} \right) \phi \left( C_{2t} \right)^{-\sigma} \tag{69}
\]

where we have used the fact that \( y_{s1t}(z) = A_{s,t} N_{1t}^s(z) \) and \( y_{s2t}(z) = A_{s,t} N_{2t}^s(z) \)

Aggregation

Household demand for flexible price and sticky price goods is given by expressions similar to equation (9), (10), (14) and (15) with \( C_i' \) and \( C_i'' \) replaced by \( C_i' \) and \( C_i'' \) where \( i = 1, 2 \). Total demand for the flexible price good is given by

\[
C_{f,t} = \gamma \left( \frac{P_{f,t}}{P_t} \right)^{-\eta} C_i + (1 + \lambda) C_s \tag{70}
\]

where \( C_i = \lambda_1 C_i' + (\lambda - \lambda_1) C_{1t} + \lambda_2 C_i'' + (1 - \lambda_2) C_{2t} \)

(71)

And the total demand for the sticky price good is given by

\[
C_{s,t} = (1 - \gamma) \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} C_i \tag{72}
\]

Market Clearing

The market for the flexible price good clears

\[
Y_{f,t} = \lambda_1 y_{f1,t} + (\lambda - \lambda_1) y_{f2,t} = C_{f,t} \tag{73}
\]

\(^5\) Those who can change prices will choose the same price while others will continue with the prices fixed earlier.
where
\[ y_{f,1,t} = A_{f,t} N_{1t}^f \quad \text{and} \quad y_{f,2,t} = A_{f,t} N_{2t}^f \]

The market for the sticky price good clears
\[ Y_{s,t} = \lambda_1 y_{s,1,t} + (1 - \lambda_2) y_{s,2,t} = C_{s,t} \tag{74} \]

where
\[ y_{s,1,t} = A_{s,t} N_{1t}^s \quad \text{and} \quad y_{s,2,t} = A_{s,t} N_{2t}^s \]

The bond market clears
\[ \lambda_1 B_t^f + \lambda_2 B_t^s = 0 \tag{75} \]

Equations (43), (44), (48), (49) of Appendix I and (56), (57), (59), (61), (62), (64), (66)-(75) expressed in terms of stationary variables define the system of equations that, combined with the monetary policy rule and exogenous stochastic processes for \( A_{f,t} \) and \( A_{s,t} \), determine the equilibrium path of the economy under this general setting.